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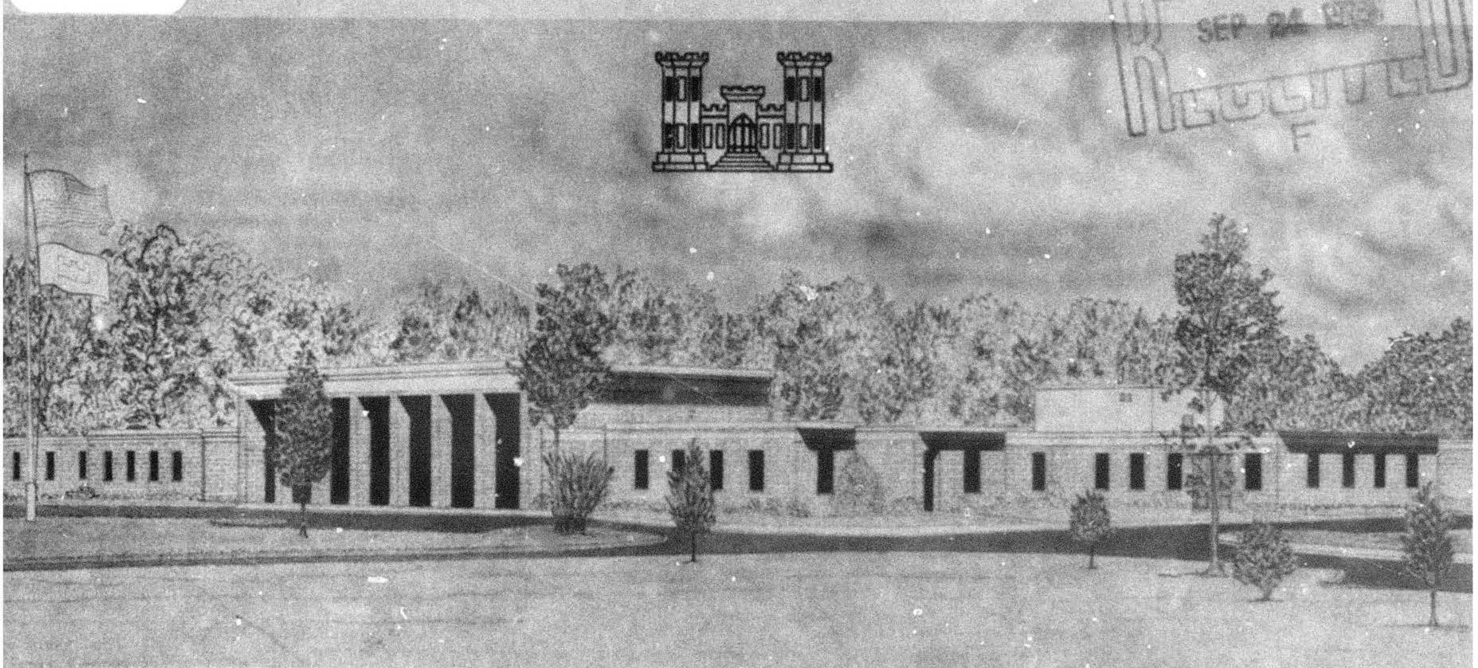
TECHNICAL REPORT M-73-5

A NUMERICAL MODEL OF THE RIDE DYNAMICS OF A VEHICLE USING A SEGMENTED TIRE CONCEPT

by

W. F. Ingram



August 1973

Sponsored by U. S. Army Materiel Command
Research, Development and Engineering Directorate

Conducted by U. S. Army Engineer Waterways Experiment Station
Mobility and Environmental Systems Laboratory
Vicksburg, Mississippi. 39180

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FOREWORD

This report was prepared by Mr. Windell F. Ingram of the Systems Software Section, Operations Branch, Automatic Data Processing Center, U. S. Army Engineer Waterways Experiment Station. The report is essentially a thesis submitted by Mr. Ingram in partial fulfillment of the requirements for the degree of Master of Science in Engineering to the Faculty of Mississippi State University, and is a study concerned with vehicle dynamics. The study described herein was conducted under the auspices of the Mobility and Environmental Systems Laboratory, Waterways Experiment Station under DA Project 1T162112A046, "Trafficability and Mobility Research," Task 03, "Mobility Fundamentals and Model Studies," under the sponsorship and guidance of the Research, Development and Engineering Directorate, U. S. Army Materiel Command. The study was accomplished under the general direction of Messrs. S. J. Knight and W. G. Shockley.

BG Ernest D. Peixotto, CE, and COL G. H. Hilt, CE, were Directors of the Waterways Experiment Station during the period of preparation and publication of this report. Mr. F. R. Brown was Technical Director.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

SYMBOL	DESCRIPTION
C_j	Force-velocity rate of the j^{th} suspension damper
cc_j	Force-velocity rate of the j^{th} tire
g	Acceleration of gravity
h	Integration time-step
I_i	Roll moment of inertia of the i^{th} axle
I_p	Vehicle body pitch moment of inertia
I_r	Vehicle body roll moment of inertia
k_j	Force-deflection rate of the j^{th} suspension spring
kk_j	Force-deflection rate of the j^{th} tire
ℓ_j	Distance from the vehicle body center of gravity to the j^{th} suspension
$\ell\ell_i$	Distance from the vehicle body center of gravity to the i^{th} axle
L_j	Distance from the vehicle body center of gravity to the j^{th} wheel
n	Number of axles
N	Number of degrees of freedom for the vehicle
W_i	Weight of the i^{th} axle
W	Weight of the vehicle body
x_i	Vertical displacement of the i^{th} axle from its static equilibrium position
xx_j	Vertical deflection of the j^{th} tire

SYMBOL	DESCRIPTION
X	Vertical displacement of the vehicle body from its static equilibrium position
θ	Pitch angle of the vehicle body
ϕ	Roll angle of the vehicle body
ϕ_i	Roll angle of the i^{th} axle

CHAPTER I: INTRODUCTION

A vehicle body considered as rigid possesses six degrees of freedom, and may experience any of six different types of motion. Motion in the direction of travel of the vehicle is related to acceleration and braking characteristics and is generally called the "performance" motion. Yawing and sideslipping are associated with steering control and are called "handling" motions. Bounce, pitch, and roll are considered to be "ride" motions of the vehicle.

In a general sense, the analysis of ground vehicle ride can be viewed as a problem involving the riding surface, the vehicle, and the vehicle occupants. A complete analysis of occupant comfort would deal not only with the physical phenomena stimulating occupants, but also with the physiological and psychological responses of the occupants. The vehicle occupants are stimulated by vehicle motions and by noises and vibrations coming from the power plant, the power transmission system, and other sources. While stimulated by the vehicle's response to steering, acceleration, and braking, the occupants' most frequent stimulation is due to the motions of bounce, pitch, and roll. These components of ride dynamics, which may be defined as the vehicle's response motions to excitations from riding surface irregularities, are the only vehicle motions considered in this study. While some researchers consider roll motions to lie in the domain of handling rather than ride characteristics, roll has been shown to contribute significantly to vertical motions of the vehicle occupants, especially in cross-country vehicles where terrain profiles along the paths of the two sides of

the vehicle can be significantly different. The effects of engine vibrations and vehicle frame flexure are not generally considered to be significant contributors to ride dynamics and will not be considered here.

Generally, the primary objective of an analysis of ride dynamics is to improve the ride characteristics of vehicles through an understanding of the mechanics of vehicle response to riding surface irregularities. However, some analyses may be for the specific purpose of determining which existing vehicle design exhibits most favorable response characteristics to a given type riding surface, thus facilitating selection of a vehicle for a particular application.

The three methods used in ride analysis are generally referred to as observation, experimental analysis, and mathematical analysis.

Of the three methods, the oldest and most obvious is that of observation. This is simply to observe the ride of the vehicle on various types of riding surfaces, make changes, and observe their influence. This has been the most widely used method and it was largely through the use of this method that the ride of our present vehicles evolved. The observation method has several drawbacks:

1. The method is subjective, depending on the observer for memory and evaluation, and does not give a numerical measurement of ride characteristics.
2. While detecting disturbing ride characteristics, the observer has no way to determine the source of the disturbance or what changes are necessary to improve the condition.
3. Any changes in vehicle design must be fully implemented in the prototype vehicle before evaluation can be made.

Experimental methods improve on the observation method by employing instrumentation to quantify the vehicle response. Modern electronic instrumentation makes it possible to measure and record many vehicle motions plotted against time. Experimental methods, which are widely used, share with the observation method the disadvantage of requiring a vehicle prototype for design evaluation. Proposed design changes can be evaluated only by construction of the changed component and implementation in the prototype. It is also necessary to construct a test track or in some manner simulate the riding surface for which the vehicle is intended.

Mathematical analyses have been made feasible by the advent of high-speed analog and digital computers. Potential advantages of mathematical analyses are

1. They provide a fundamental understanding of the interrelation of the various components of the vehicle. This provides a better understanding of the influence of a design change.
2. The effect of a proposed design change can be estimated without building a prototype.
3. Comparison between many existing vehicle designs on varying riding surfaces can be made by changing the parameters associated with the mathematical model.

Experimental analyses are often used in conjunction with and for verification of mathematical models.

Historical Background

Attempts at analytical approaches to vehicle dynamics date back to shortly after construction of the first automobile. One of the first by Rowell (21)* in 1922 dealt with undamped, free vibrations.

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*Numbers in parentheses refer to similarly numbered entries in the Bibliography.

In 1926 Guest (9) developed a system of differential equations to describe the vibration motion of a vehicle. His equations included effects of body bounce, body pitch, and axle bounce in a system consisting of body mass, suspension springs, axle masses, and tire vibration for body bounce and pitch to the motions felt by passengers located in different parts of the vehicle. In 1934 Olley (18) extended the work of Guest to include independent suspension systems. Through calculations and experiments he determined the effects of changes to a vehicle's design parameters (suspension and damping rates) on the vehicle's periods of free vibration in body bounce and pitch. He also determined that ride comfort is not solely a matter of opinion, but that a definite determination of the motion functions that produce discomfort could be made with variation being small between individuals. His work made it possible to visualize clearly many of the problems associated with vehicle springing. In 1935 James, et al. (11), using the analysis by Guest as a basis, developed the first method for obtaining the pitch moment of inertia of a vehicle body experimentally. This was a significant contribution since an accurate value of the moment of inertia is essential to mathematical analysis of ride. These early attempts at ride analysis, while producing considerable insight into the problem, soon became too mathematically complex for solution by manual methods and it became apparent that some sort of simulation of the vehicle would be necessary.

In 1943 Schilling and Fuchs (23) built a mechanical differential analyzer to solve the nonlinear differential equation associated with

a single mass, single spring damper system. Their analyzer permitted the inclusion of a nonlinear characteristic for the shock absorber and was used to determine the transient vertical accelerations that might be experienced by a vehicle passenger. They demonstrated how these accelerations could be altered drastically by changes in the characteristics of the shock absorber. This analyzer was considered to be the first analog computer used in suspension analysis and was a significant advance to the state of the art.

In the early 1950's there was considerable work directed toward passenger car suspension analysis in the automotive industry as a result of the advent of the analog computer. A significant advance was made in 1953 by Jeska (12) with the presentation of a model that included vertical motions of the front and rear wheels together with the bounce and pitch of the body. The unique contribution here was that the analysis included an actual road profile, measured by a photographic technique, as a driving function to the model.

In 1955 Bodeau, et al. (2), presented a very good general discussion of ride analysis and a detailed analysis using a nine-degree-of-freedom model to describe a passenger car. This work, which included both an experimental analysis and a mathematical analysis using an analog computer, attempted to determine human response characteristics in terms of passenger comfort limits. Up to this point, little had been learned regarding passenger tolerance to vibrations.

Kohr (10) in 1960 constructed a servo-controlled motion simulator which could seat two occupants and which followed motions

determined by an analog computer. This "ride simulator" included seven degrees of freedom, i.e., body bounce, roll, and pitch, plus bounce and roll of each of the two axles. A road profile was recorded on magnetic tape and then "played" into the analog computer as input disturbance to the model suspension system. The simulator was occupied by a driver and passenger to provide a means of determining the effects of the vibrational characteristics of humans. The ride simulator was an important advancement for the automotive industry because it enabled improvements in passenger car ride to be made with a minimum of time and effort.

Up until the late 1950's most of the research in ride dynamics had been directed at on-the-road vehicles such as passenger cars. However, in this case most of the severe effects of ride motions had been overcome by altering the environment rather than the vehicle; i.e., we have designed and built smooth roads that produce rather minor disturbances to the suspension system as compared with cross-country locomotion.

In the late 1950's the military began to recognize the significance of the ride dynamics problem in off-road mobility. For off-road locomotion to achieve maximum effectiveness, ride motions due to terrain roughness must cause neither mechanical failure nor human response that reduces the average velocity of the vehicle. Mechanical failure of military vehicles due to terrain roughness is not common, thus the military driver in an emergency will always drive to the limits of his tolerance to vibratory motions. Analysis of ride stimuli to the driver and crew in this environment bears little resemblance to determination of comfort

levels in the usual automotive sense. The objective is to determine limiting speeds for cross-country vehicles in terms of the vehicle's design parameters, the terrain roughness, and human tolerance to vibratory motions.

As a result of this military awareness of ride dynamics, mathematical models to assess the influence of various design alternatives on the performance of military vehicles have become important. Greater requirements for ground mobility have produced requirements for new vehicles. There is no longer time for expensive "cut and try" methods in search of new designs. Technology has increased the number of possible engineering choices to the extent that exploratory building and testing of all the alternatives is impossible. This has led to the recognition that mathematical models of terrain-vehicle systems are necessary to explore the possible choices and select a rational optimum.

Thus the 1960's saw the birth of several studies of off-road vehicle mobility, including not only analysis of ride dynamics, but also analysis of trafficability and maneuverability. Van Deusen (24) conducted a comprehensive study of the ride dynamics aspect of off-road mobility including a theoretical treatment of the dynamic behaviour of vehicles and an analysis of human response to vehicle vibration. Pradko, et al. (20), conducted basic research on the effects of vehicle vibrations on man. On the assumption that ride comfort is strongly influenced by vertical acceleration, he developed quantities to describe human dynamic tolerances. In 1965 the FMC Corporation (7) conducted a study for the Waterways Experiment Station (WES) to determine the feasibility of using a digital computer

to evaluate the ride dynamics of a vehicle traversing a hard-surface terrain. It was this FMC study that provided a starting point for the author's work on this thesis. In 1966 Cornell Aeronautical Laboratory (5) began a comprehensive study of ground mobility, the result of which included a sophisticated vehicle dynamics model using a digital computer. The model, however, was not verified against experimental results. This reference contains a comprehensive list of all ground mobility studies conducted by and for the U. S. Government in recent years. The efforts in off-road mobility research, both completed and in progress, are extensive.

Two basic approaches have been taken in attempting to interpret ride vibrations and correlate them with passenger response and terrain characteristics. The deterministic approach uses actual terrain or obstacle displacement records as a function of time as input to the model and produces information concerning vehicle motion at each instant of time. This allows determination of maximum accelerations and displacements, etc. The stochastic approach involves describing the terrain profile statistically and performing a statistical analysis of the response. The output of a stochastic analysis consists of statistical quantities associated with the vehicle motions. Although more investigations have been made using the deterministic approach, emphasis has apparently shifted to stochastic techniques.

Virtually all models developed use either an analog computer or a digital computer in the analysis. The determination of which is used is related to the nature of the study and the computing equipment available to the researcher. Generally, the analog approach

is more limited in number of degrees of freedom than the digital approach since many organizations interested in vehicle ride dynamics do not have a large scale analog computer capable of including all the degrees of freedom desired in a complex vehicle simulation. Digital computers, though readily available in sizes capable of solving many degree of freedom vehicle simulations, do not provide the speed of solution as does the analog and often involve long program run-times for representative terrain lengths.

Purpose

The general purpose of this study was to provide an improved capability for simulating vehicle ride motions felt by the driver. The specific objectives encompassed by the study were essentially three. First was the development of a practicable, efficient digital computer simulation model of the three-dimensional ride dynamics of a single-hull vehicle with an arbitrary number of axles. A background for this effort was provided by computerized models developed by FMC Corporation and by the WES. The second objective was validation of the model by comparison of model responses to field test measurements for a representative vehicle. The final objective was to investigate the effects of numerical integration step-size on the model responses and to formulate criteria for dynamic management of the step-size to achieve optimum model performance.

Scope

The effort consisted entirely of developing, implementing, and evaluating a digital computer simulation. Several field tests for a representative vehicle were simulated and comparisons are presented in the form of graphs of measured and simulated vehicle responses. The effects of integration step-size are presented in the form of graphs comparing simulated vehicle responses using various integration intervals. Criteria for management of the integration interval are established.

The scope of the study did not include the execution of field tests or the measurement of vehicle parameters. The field test data were provided by the Mobility Research Branch (MRB) of the WES and the vehicle parameters used in the simulation came from the MRB and the FMC Corporation. The data were accepted as being as reliable as any that were available during the course of this study.

CHAPTER II: THE MATHEMATICAL MODEL

The Differential Equations of Motion

The ride motions of a generalized three-dimensional vehicle traversing an unyielding riding surface can be described by writing a set of simultaneous differential equations relating the accelerations of the vehicle components to the spring and damping forces applied by the pneumatic tires and suspensions (springs and shock absorbers). Considering the vehicle body and axles as rigid masses, the motions to be included are body bounce, body pitch, body roll, axle bounce, and axle roll. No fore and aft or transverse vibratory motions are to be considered here. By viewing the vehicle body and axles as displaced from their static equilibrium position (Figure 1)*, we can identify the spring and viscous damper forces contributing to body bounce as shown below:

Force produced by vertical displacement of the body:

$$-X \sum_{j=1}^{2n} k_j \qquad (1)$$

Force produced by vertical motion of the body:

$$-\dot{X} \sum_{j=1}^{2n} c_j \qquad (2)$$

Force produced by body pitch displacement:

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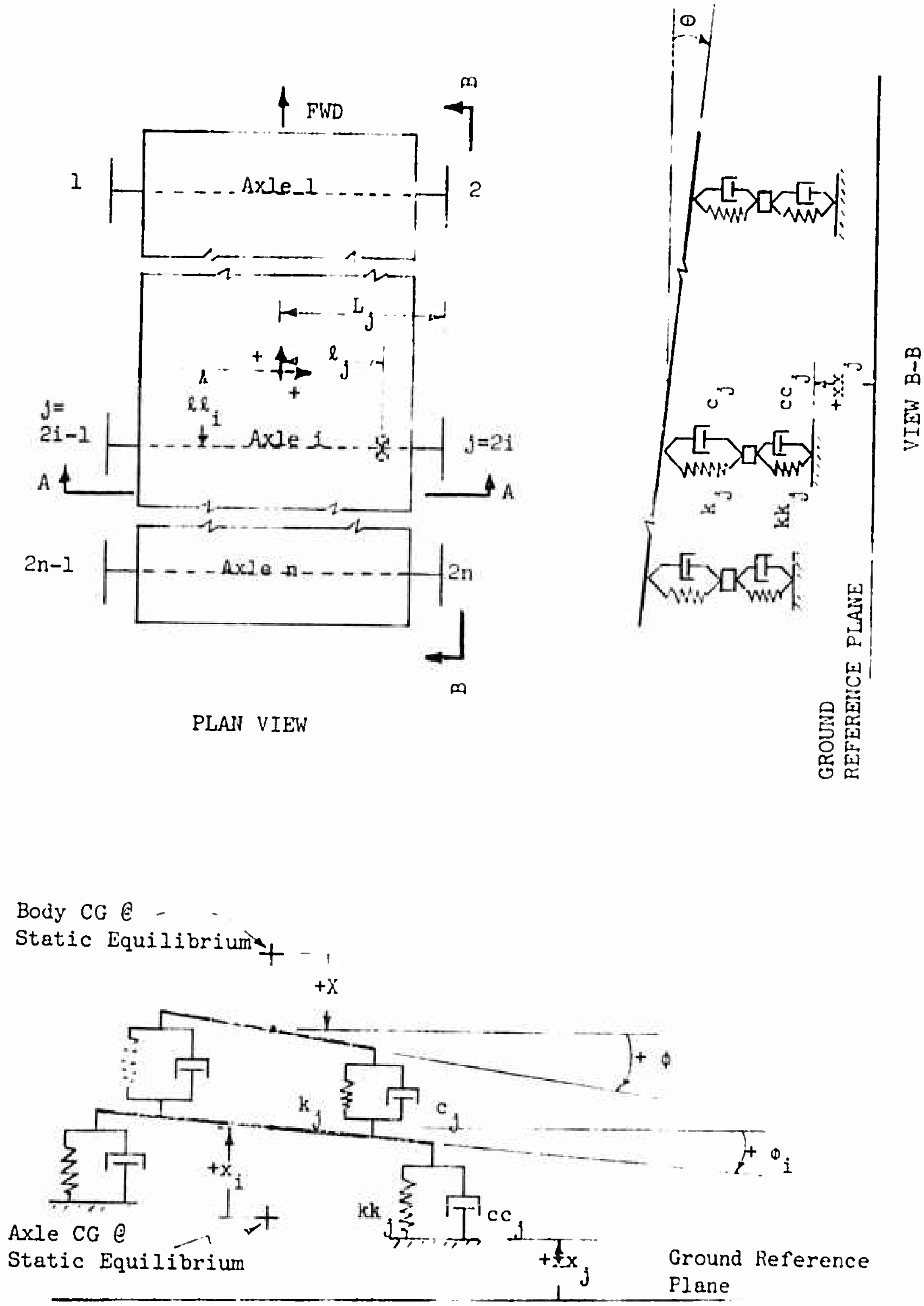
*Each tire is depicted in Figure 1 as a single spring-damper assembly for convenience only. As actually modeled, stiffness of each tire is represented by many radially spaced springs as will be shown on the following pages.



VIEW A-A

Figure 1. Schematic of the Vehicle Model

$$-\sin\theta \sum_{i=1}^{n} \sum_{j=2i-1}^{2i} k_j \ell \ell_i \qquad (3)$$

Force produced by body pitch motion:

$$-\dot{\theta} \cos\theta \sum_{i=1}^{n} \sum_{j=2i-1}^{2i} c_j \ell \ell_i \qquad (4)$$

Force produced by body roll displacement:

$$-\sin\phi \sum_{j=1}^{2n} k_j \ell_j \qquad (5)$$

Force produced by body roll motion:

$$-\dot{\phi} \cos\phi \sum_{j=1}^{2n} c_j \ell_j \qquad (6)$$

Force produced by vertical displacement of the axles:

$$-\sum_{i=1}^{n} \sum_{j=2i-1}^{2i} x_i k_j \qquad (7)$$

Force produced by vertical motion of the axles:

$$-\sum_{i=1}^{n} \sum_{j=2i-1}^{2i} \dot{x}_i c_j \qquad (8)$$

Force produced by axle roll displacement:

$$+\sum_{i=1}^{n} \sum_{j=2i-1}^{2i} \sin \phi_i k_j \ell_j \qquad (9)$$

Force produced by axle roll motion:

$$+\sum_{i=1}^{n} \sum_{j=2i-1}^{2i} \dot{\phi}_i \cos\phi_i c_j \ell_j \qquad (10)$$

Summing suspension forces and equating to the body accelerating force yields: (11)

$$\frac{\ddot{X}W}{g} = -X\sum_{j=1}^{2n} k_j - \dot{X}\sum_{j=1}^{2n} c_j - \sin\theta\sum_{i=1}^{n}\sum_{j=2i-1}^{2i} k_j \ell\ell_i - \dot{\theta}\cos\theta\sum_{i=1}^{n}\sum_{j=2i-1}^{2i} c_j \ell\ell_i -$$

$$\sin\phi\sum_{j=1}^{2n} k_j \ell_j - \dot{\phi}\cos\phi\sum_{j=1}^{2n} c_j \ell_j - \sum_{i=1}^{n}\sum_{j=2i-1}^{2i} x_i k_j - \sum_{i=1}^{n}\sum_{j=2i-1}^{2i} \dot{x}_i c_j +$$

$$\sum_{i=1}^{n}\sum_{j=2i-1}^{2i} \sin\phi_i k_j \ell_j + \sum_{i=1}^{n}\sum_{j=2i-1}^{2i} \dot{\phi}_i \cos\phi_i c_j \ell_j$$

Similarly the differential equation describing body pitch can be written by summing the pitching moments about the body center of gravity. (12)

$$\frac{\ddot{\theta}I_p}{\cos\theta} = -X\sum_{i=1}^{n}\sum_{j=2i-1}^{2i} k_j \ell\ell_i - \dot{X}\sum_{i=1}^{n}\sum_{j=2i-1}^{2i} c_j \ell\ell_i - \sin\theta\sum_{i=1}^{n}\sum_{j=2i-1}^{2i} k_j \ell\ell_i^2$$

$$-\dot{\theta}\cos\theta\sum_{i=1}^{n}\sum_{j=2i-1}^{2i} c_j \ell\ell_i^2 - \sin\phi\sum_{i=1}^{n}\sum_{j=2i-1}^{2i} k_j \ell_j \ell\ell_i - \dot{\phi}\cos\phi\sum_{i=1}^{n}\sum_{j=2i-1}^{2i} c_j \ell_j \ell\ell_i$$

$$-\sum_{i=1}^{n}\sum_{j=2i-1}^{2i} x_i k_j \ell\ell_i - \sum_{i=1}^{n}\sum_{j=2i-1}^{2i} \dot{x}_i c_j \ell\ell_i + \sum_{i=1}^{n}\sum_{j=2i-1}^{2i} \sin\phi_i k_j \ell_j \ell\ell_i +$$

$$\sum_{i=1}^{n} \sum_{j=2i-1}^{2i} \dot{\phi}_i \cos\phi_i c_j \ell_j \ell\ell_i$$

Coupled with the pitch and bounce equations is an equation describing the rolling motion of the vehicle body. This is obtained by summing the rolling moments about the body center of gravity.

$$\frac{\ddot{\phi} I_r}{\cos\phi} = -X \sum_{j=1}^{2n} k_j \ell_j - \dot{X} \sum_{j=1}^{2n} c_j \ell_j - \sin\theta \sum_{i=1}^{n} \sum_{j=2i-1}^{2i} k_j \ell_j \ell\ell_i \tag{13}$$

$$-\dot{\theta}\cos\theta \sum_{i=1}^{n} \sum_{j=2i-1}^{2i} c_j \ell_j \ell\ell_i - \sin\phi \sum_{j=1}^{2n} k_j \ell_j^2 - \dot{\phi}\cos\phi \sum_{j=1}^{2n} c_j \ell_j^2$$

$$-\sum_{i=1}^{n} \sum_{j=2i-1}^{2i} x_i k_j \ell_j - \sum_{i=1}^{n} \sum_{j=2i-1}^{2i} \dot{x}_i c_j \ell_j + \sum_{i=1}^{n} \sum_{j=2i-1}^{2i} \sin\phi_i k_j \ell_j^2$$

$$+\sum_{i=1}^{n} \sum_{j=2i-1}^{2i} \dot{\phi}_i \cos\phi_i c_j \ell_j^2$$

Each axle assembly requires two differential equations, one describing axle bounce and one describing axle roll, to couple with the three body equations. The equation describing

axle bounce is obtained by summing forces from the suspensions and tires connected to an axle. The axle bounce equation is written as:

$$\frac{\ddot{x}_i w_i}{g} = -X\sum_{j=2i-1}^{2i} k_j - \dot{X}\sum_{j=2i-1}^{2i} c_j - \sin\theta \ell\ell_i \sum_{j=2i-1}^{2i} k_j - \dot{\theta}\cos\theta \ell\ell_i \sum_{j=2i-1}^{2i} c_j - \tag{14}$$

$$\sin\phi \sum_{j=2i-1}^{2i} k_j \ell_j - \dot{\phi}\cos\phi \sum_{j=2i-1}^{2i} c_j \ell_j - x_i \sum_{j=2i-1}^{2i} (k_j + kk_j) - \dot{x}_i \sum_{j=2i-1}^{2i} (c_j + cc_j) +$$

$$\sin\phi_i \sum_{j=2i-1}^{2i} (k_j \ell j + kk_j L_j) + \dot{\phi}_i \cos\phi_i \sum_{j=2i-1}^{2i} (c_j \ell_j + cc_j L_j) + \sum_{j=2i-1}^{2i} xx_j kk_j$$

$$+ \sum_{j=2i-1}^{2i} \dot{xx}_j cc_j$$

The general equation for axle roll is obtained by summing the rolling moments about the axle center of gravity.

$$\frac{\ddot{\phi}_i I_i}{\cos\phi_i} = + X \sum_{j=2i-1}^{2i} k_j \ell_j + \dot{X} \sum_{j=2i-1}^{2i} c_j \ell_j + \sin\theta \ell\ell_i \sum_{j=2i-1}^{2i} k_j \ell_j + \tag{15}$$

$$\dot{\theta}\cos\theta \ell\ell_i \sum_{j=2i-1}^{2i} c_j \ell_j + \sin\phi \sum_{j=2i-1}^{2i} k_j \ell_j^2 + \dot{\phi}\cos\phi \sum_{j=2i-1}^{2i} c_j \ell_j^2 + x_i \sum_{j=2i-1}^{2i} (k_j \ell_j + kk_j L_j)$$

$$+ \dot{x}_i \sum_{j=2i-1}^{2i} (c_j \ell_j + cc_j L_j) - \sin\phi_i \sum_{j=2i-1}^{2i} (k_j \ell_j^2 + kk_j L_j^2)$$

$$- \dot{\phi}_i \cos\phi_i \sum_{j=2i-1}^{2i} (c_j \ell_j^2 + cc_j L_j^2) - \sum_{j=2i-1}^{2i} xx_j kk_j L_j - \sum_{j=2i-1}^{2i} \dot{xx}_j cc_j L_j$$

The coupled set of second order differential equations describes all the components of vehicle motion associated with ride dynamics, i.e., body bounce, body pitch, body roll, axle bounce, and axle roll. Thus for an n-axled vehicle the system described has N degree of freedom where

$$N = 2n + 3$$

It includes all the nonlinearities associated with large rotational motions, i.e., $\sin\theta$, $\cos\theta$, $\sin\phi$, $\cos\phi$, etc.

The Segmented Tire Concept

Many vehicle dynamics models use a representation of the pneumatic tire-riding surface contact mechanism similar to that shown in Figure 2. The flexure property of the tire is represented by a single spring element which may have either linear or nonlinear properties. Dissipation of energy is accommodated by a viscous damper which may also be linear or nonlinear. Using this technique, the single point of contact is deflected by changes in elevation along the riding surface profile.

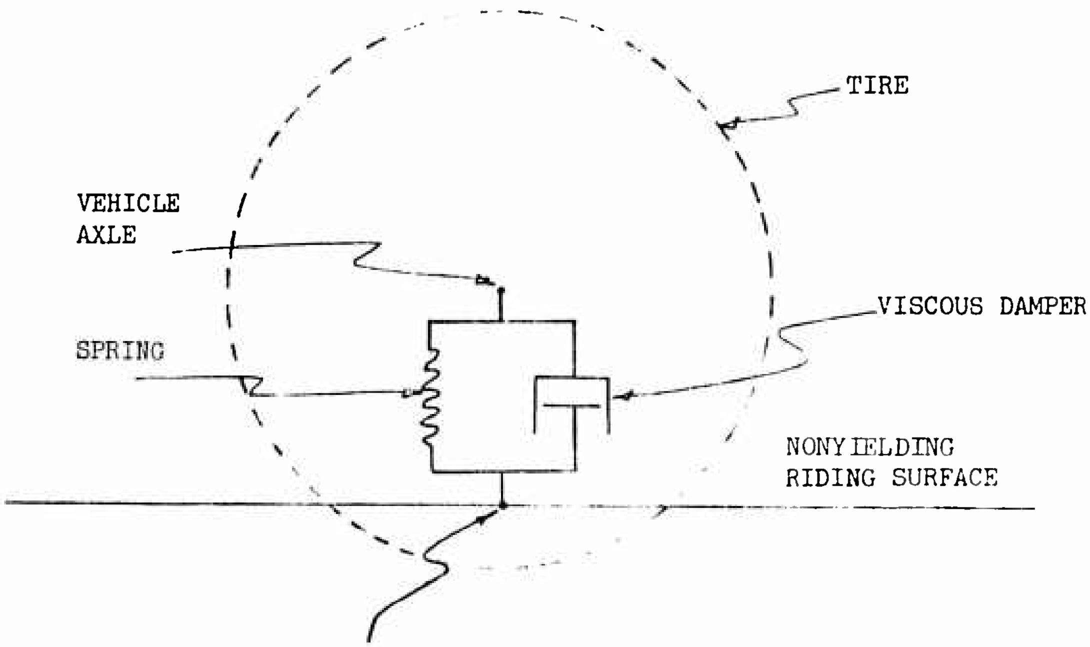


Figure 2. Depiction of a Point-Contact Concept

This point-contact method may be adequate in many cases where the riding surface is relatively smooth and there are no abrupt changes in surface slope. For on-the-road vehicle models this method may be sufficiently representative of the contact mechanism while providing a relatively simple implementation.

For off-road models where large surface irregularities, obstacles, and abrupt slope changes may be encountered, the point-contact concept is a questionable representation of the tire-terrain interface. It provides none of the "enveloping" characteristics of a pneumatic tire. That is, small terrain features cannot produce deflections of small areas of the tire carcass. Every deflection of the point of contact produces forces representative of a deflection of the entire contact surface by the same magnitude. Figure 3 shows an example of this condition. The small round obstacle produces a deflection at the contact point that produces forces of the same magnitude as would be produced by a deflection of the entire contact surface as shown by the solid line. The obstacle would actually be somewhat enveloped by the tire, the tire surface being more closely approximated by the dashed line.

Seeking a more realistic tire simulation method, A. S. Lessem (13) of the WES Mobility Research Branch conceived a variable-area contact mechanism that simulates tire envelopment properties and provides for transmission of horizontal as well as vertical forces through the tire. The flexure property of the tire is represented by many radial springs spaced throughout an "active region" of the tire, i.e., that area of the tire that may contact the riding

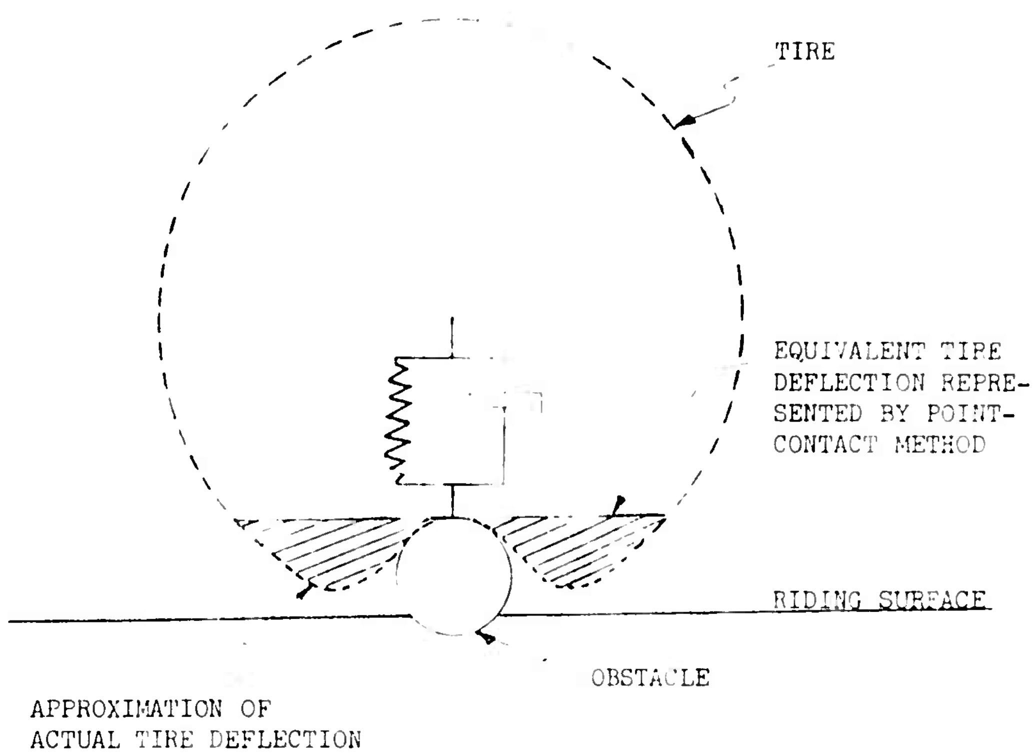


Figure 3. Example of Excess Deflection Produced by Point-Contact Method

surface (Figure 4). Each spring, representing one "segment" of the tire carcass, was considered linear with spring coefficient K being the same for each segment. Individual springs of the segmented tire are deflected by the riding surface profile; each spring deflection produces a force vector; summing these force vectors produces a resultant force vector representative of the spring force transmitted to the vehicle axle. As forward vehicle motion is simulated by moving the nonrotating active region relative to the riding surface profile, surface irregularities produce spring deflections in sequence from the leading boundary to the trailing boundary of the active region (Figure 5).

To implement this method of simulating tire deflection, it was necessary to first establish a segment spring constant (K) for each tire at each pressure under consideration. An experimentally determined static force-deflection relationship for the tire was obtained. Then a single point on the force deflection curve was chosen for the purpose of computing K. It was found that in most cases a vertical deflection of approximately one inch was an optimum for this purpose. From Figure 4 static equilibrium requires that

$$F = 2 \sum_{i=1}^{n} K\Delta_i \cos\phi_i \qquad (16)$$

Where:

F = Vertical load at the chosen deflection, lb;

K = Segment spring coefficient, lb/in;

Δ_i = Effective radial deflection, i^{th} segment, in;

ϕ_i = Force direction angle, i^{th} segment, radians;

n = Number of segments each side of tire center line.

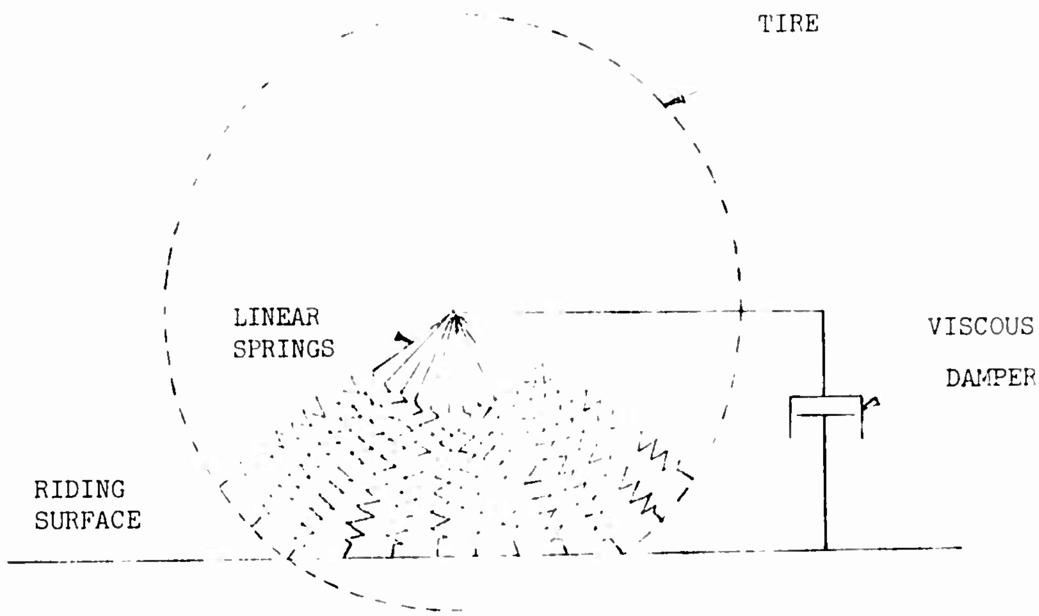


Figure 4. Schematic of Segmented Tire

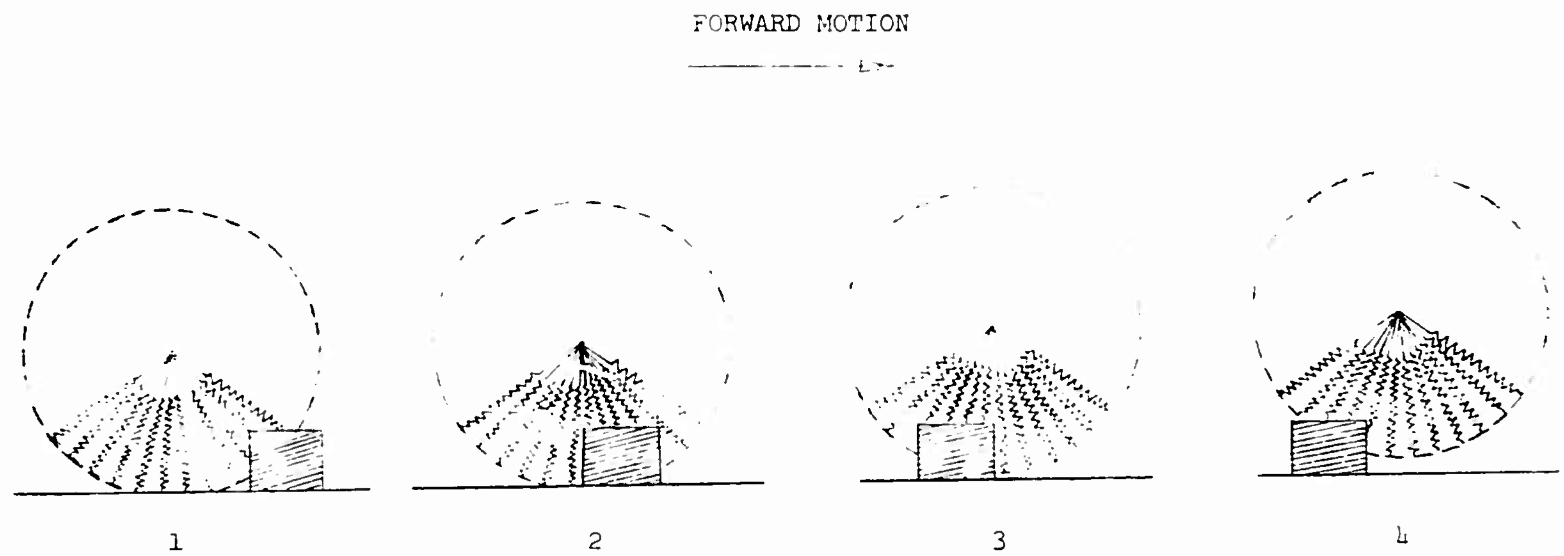


Figure 5. Obstacle Passing Through the Active Region of the Segmented Tire

Then K can be computed as

$$K = \frac{F}{2\sum_{i=1}^{n} \Delta_i \cos\phi_i}$$

Figure 6 shows an example of the actual measured force-deflection curve for a specific tire and the force-deflection curve resulting from a segmented tire representation using twelve segments spaced at ten-degree intervals. Note that the modeled deflection curve is piece-wise linear with a point of discontinuity where, as deflection increases, additional segments are included in the resisting force.

This concept was first modeled dynamically using an analog computer to simulate the motion of a single military vehicle tire. The validity of the results was verified by experimental test at the WES.

In 1968 this mechanism for simulating the tire-riding surface interface was incorporated into the FMC vehicle dynamics digital computer program by J. F. Smith (16) of the WES Automatic Data Processing Center. Smith's digital implementation of the segmented tire concept is essentially the tire mechanism employed herein. The writer has made modifications to the tire subprogram for the purpose of eliminating some oversights in the original code and to increase the speed of computation.

Incorporation of the segmented tire concept into the general vehicle dynamics model necessitates modification of the differential equations describing axle bounce and roll. In equation 14, the term

$$\sum_{j=2i-1}^{2i} xx_j kk_j \qquad (18)$$

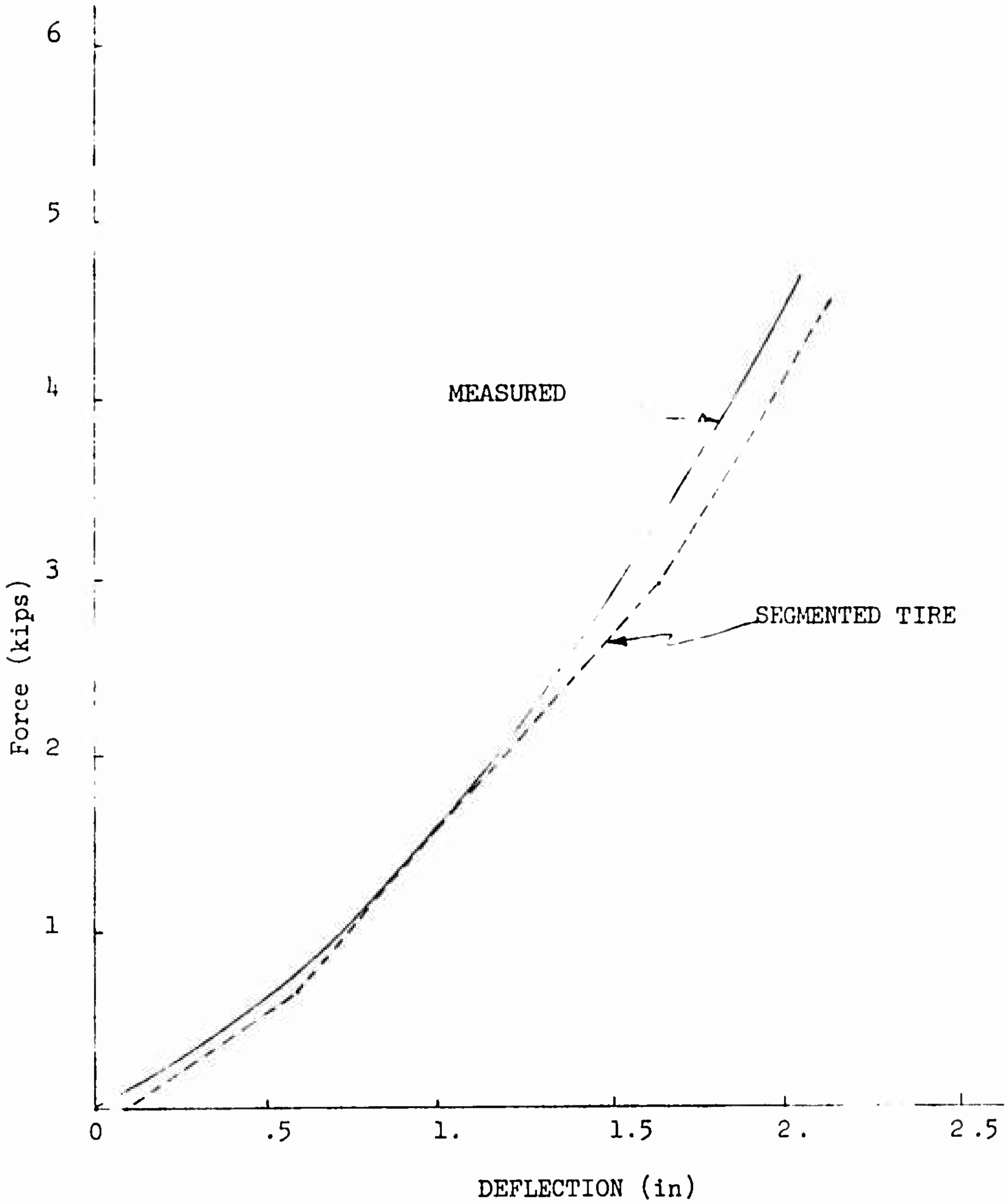


Figure 6. Force Deflection Relationship for 9.00-16 Tire @ 45 psi

is replaced by

$$\sum_{j=2i-1}^{2i} F_j \tag{19}$$

and in equation 15, the term

$$\sum_{j=2i-1}^{2i} xx_j kk_j L_j \tag{20}$$

is replaced by

$$\sum_{j=2i-1}^{2i} F_j L_j \tag{21}$$

where F_j is the vertical component of the spring force vector transmitted by the j^{th} wheel to its axle. F_j is computed by summing the force vectors contributed by each segment's spring deflection. The spring deflections are determined by computing the point of intersection of each segment spring with the riding surface contacting the tire.

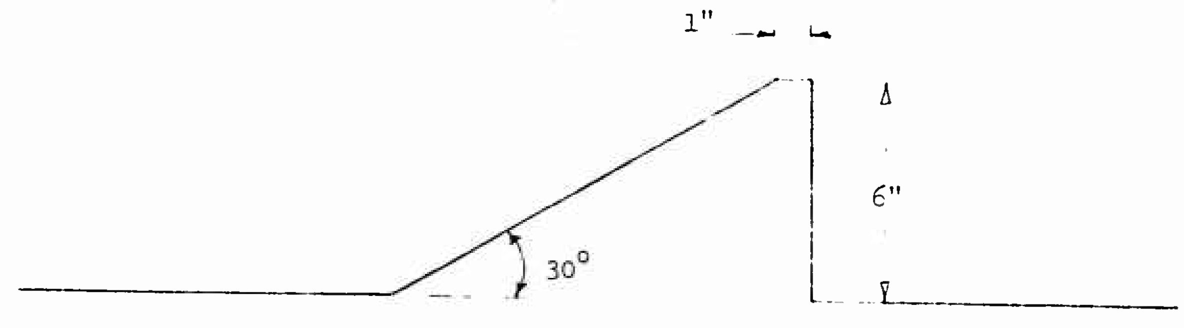
Note again that the force-deflection characteristics of the segmented tire model include many discontinuities in stiffness rates as the tire carcass flexes. Effects of this will be discussed later.

While no attempt has been made to incorporate damping properties into each tire segment as with spring stiffness, tire damping is included in the digital model as a single viscous damper responding to motion of the wheel hub (as shown in Figure 2 with the point-contact method).

Representation of the Riding Surface Profile

The hard-surface profile which serves as a displacement function for the tire spring, thus in essence a forcing function to the system of masses, springs, and dampers representing the vehicle, is represented by a series of coordinates connected by straight line segments. Elevations along the surface are determined by simple linear interpolation between the points. Examples of a typical obstacle test profile and a typical segment of cross-country terrain are shown in Figure 7.

One might consider linear interpolation between profile points to be a crude method of describing a terrain profile for the purpose of simulating vehicle traversal. A part of the FMC study was directed at determining practical methods for describing such profiles. A Fourier series was used as one method of mapping a terrain profile, and the method was found to be accurate but very inefficient due to the increase in computer time required to generate and process the Fourier series. It was found that use of linear interpolation between profile points produced sufficient accuracy to make this a valid approach provided that the profile point spacing was not excessive. The linear interpolation method represents the absolute minimum in computer time that would be required by any method.



OBSTACLE

SEGMENT OF CROSS-COUNTRY TERRAIN (SCALE: 1" = 50")

Figure 7. Profiles of ar Obstacle and a Segment of Cross-Country Terrain

CHAPTER III: SOLUTION OF THE EQUATIONS OF MOTION

Digital simulation of a dynamic system such as the nonlinear system described by the equations of motion for the generalized solid axle vehicle requires a discrete representation of the continuous system, i.e., a discrete state model. Given an initial system state, the next system state (after some time interval h) is to be determined. This makes the system simulation a problem in numerical integration.

Numerical integration of systems of second-order differential equations of this type is commonly handled by first expressing the second-order equations as a system of first-order equations to which general numerical integration techniques are applicable. Letting y_i represent a general displacement of a vehicle component, the equations of motion can be expressed in general terms as:

$$\ddot{y}_i = f_i(t, y_1, \dot{y}_1, y_2, \dot{y}_2, \ldots, y_n, \dot{y}_n), i=1\ldots n \tag{22}$$

where i denotes the i^{th} differential equation and n is the number of degrees of freedom, i.e., the number of differential equations in the system. Conversion to a system of first-order equations is accomplished by substituting z_i for $\dot{y}_i$. Thus the equations of motion can be expressed as:

$$\begin{aligned} \dot{y}_i &= z_i \\ \dot{z}_i &= f_i(t, y_1, z_1, y_2, z_2, \ldots y_n, z_n), i=1\ldots n \end{aligned} \tag{23}$$

In practice this means that computed accelerations ($\dot{z}_i$) are used to solve for velocities (z_i) which are in turn integrated numerically to produce displacements (y_i). Given the initial values of displacement and velocity, this is a practicable means of integrating dynamics equations.

Given the system of first-order equations, numerical integration is usually accomplished by one of two basic approaches. One is a two-step predictor-corrector procedure requiring the use of the present system state plus several previous states. The second approach is a one-step procedure which requires computation of several intermediate values, but requires use of the present state only, e.g., the Runge-Kutta methods.

A cursory review of the literature might lead one to believe there is an abundance of good numerical integration techniques and that the choice of method might be somewhat arbitrary. However, selection of the integration technique plays a large part in the overall system simulation. Desired attributes of the selected method include speed, accuracy, stability, and convenience of use. Maximum speed can be obtained through selecting a fast method and using a large time step; acceptable accuracy may require slower, more sophisticated methods using a much smaller time step.

Fourth-order Runge-Kutta methods are among the most popular and widely used methods of today. These methods

generally have the following advantages:

1. They are self-starting, i.e., only one known system state is required to initiate the procedure.
2. They do not usually exhibit numerical instability using properly chosen step sizes.
3. Good accuracy is usually achieved using properly chosen step sizes.
4. Relatively small amounts of core memory are required for processing.
5. They are convenient to use, being particularly amenable to general purpose subroutines.

Disadvantages generally include:

1. No estimate of accuracy.
2. No way of knowing if the step size is adequate without expensive reruns at various step sizes.
3. Requirement for four derivative evaluations per integration step compared with only two using the fourth-order predictor-corrector methods.

The Runge-Kutta-Merson Method

The Runge-Kutta-Merson (RKM) method attempts to negate some of the disadvantages associated with Runge-Kutta methods in general by using a fifth derivative evaluation to provide an estimate of discretization error to be used for automatic step-size adjustment. In general terms the Merson formulae are:

$$y_i^{j+1} = y_i^j + (P_{1i} + 4P_{4i} + P_{5i})/2 + O(h^5) \tag{24}$$

where

$$P_{1i} = hf_i\{t^j, y_1^j, y_2^j, \ldots, y_n^j\}\ /3 \tag{25}$$

$$P_{2i} = hf_i\{t^j + (h/3), y_1^j + P_{11}, y_2^j + P_{12}, \ldots, y_n^j + P_{1n}\}/3 \tag{26}$$

$$P_{3i} = hf_i\{t^j + (h/3),\ y_1^j + (P_{11}/2)+(P_{21}/2), y_2^j \tag{27}$$

$$+ (P_{12}/2) + (P_{22}/2), \ldots, y_n^j + (P_{1n}/2)+(P_{2n}/2)\}/3$$

$$P_{4i} = hf_i\{t^j + (h/2),\ y_1^j + (3P_{11}/8) + (9P_{31}/8),\ y_2^j \tag{28}$$

$$+ (3P_{12}/8) + (9P_{32}/8), \ldots, y_n^j + (3P_{1n}/8) + (9P_{3n}/8)\}/3$$

$$P_{5i} = hf_i\{t^j + h, y_1^j + (3P_{11}/2) - (9P_{31}/2) + 6P_{41}, y_2^j + (3P_{12}/2) \tag{29}$$

$$- (9P_{32}/2) + 6P_{42}, \ldots, y_n^j + (3P_{1n}/2) - (9P_{3n}/2) + 6P_{4n}\}/3$$

$O(h^5)$ indicates that the truncation error is of the order h^5 as $h \to 0$. This term is not a part of the calculations, but merely indicates that this is a fourth order method. An approximation of the error of integration (truncation error) is provided by

$$e_i = \{P_{1i} - (9P_{3i}/2) + 4\ P_{4i} - (P_{5i}/2)\}/5 \tag{30}$$

which is generally used as a basis for automatic step-size adjustment. In the equations above, t is the independent variable, the subscript i refers to the i^{th} differential equation, the superscript j refers to the previous system state just calculated, while j + 1 is the system state to be calculated, and n is the number of equations in the system.

As with all Runge-Kutta formulae, Merson's equations were derived by using Taylor series expansions and algebraic manipulations to match coefficients of terms. The process always results in more unknowns than equations, requiring an arbitrary selection of one or more constants. This arbitrary selection of constants has resulted in the many variations of the basic Runge-Kutta method. Merson's unique contribution was in his determination of an approximation of the truncation error inherent in his formalae.

The RKM process can be viewed in geometric terms as follows. For the curve associated with the i^{th} differential equation, compute the slope $\{P_{1i}/(h/3)\}$ at the point (y_i^j, t^j) and, using it, advance one-third step forward and compute the slope $\{P_{2i}/(h/3)\}$ there. Using the average of the first and second slopes computed, start again at (y_i^j, t^j) and advance one-third step forward and again sample the slope $\{P_{3i}/(h/3)\}$. Then starting again at (y_i^j, t^j) and using a weighted average of the first and third slopes computed, advance one-half step forward and compute the slope $\{P_{4i}/(h/3)\}$ at the half interval point. Finally using a weighted average of the first, third, and fourth slopes computed, advance forward a full time step from (y_i^j, t^j) and compute the slope $\{P_{5i}/(h/3)\}$ at the point (y_i^{j+1}, t^{j+1}). Having computed all the intermediate derivatives, establish the point (y_i^{j+1}, t^{j+1}) by advancing one full time step from (y_i^j, t^j) using a weighted average of the slopes computed at the beginning, the midpoint, and the end of the time interval. The error estimate e_i (equation 30) can be viewed geometrically as an evaluation of the variation of the slopes at the intermediate point, i.e., the greater the variation in slopes

over a time interval, the greater the error introduced in that time step. Hence, it seems reasonable from a geometric point of view to use some evaluation of intermediate slope differences such as equation 30 as an indicator of the need for automatic interval adjustment.

Management of the Numerical Integration Step-Size

The primary requirements for acceptable performance of Merson's or any other numerical integration technique are speed, accuracy, and stability. All three factors are directly coupled to the choice of the time increment employed. Since most of the computing time involved in simulation of vehicle dynamics (or any other dynamic system) is used in evaluating the derivatives, the computing cost is directly proportional to the number of derivative evaluations required, hence inversely proportional to the size of the time-step chosen, e.g., halving the step-size doubles the computing cost. In complex systems such as is being simulated here, the computing time required per time-step precludes choosing a "safe," i.e. very small, constant time-step to assure accuracy and stability. Such a choice would make the computing costs so high as to render simulations of an effective time duration impracticable. Choosing a large constant time-step to achieve reasonable computing costs can result in unpredictable performance or unreliable results. Thus proper management of the time-step-size, automatically by the computer program, can mean the difference between an unreliable, unpredictable, or impracticable model and a workable model producing acceptable accuracy at an acceptable cost.

What constitutes "management" of the integration time-step? Ideally, the objective is to produce accurate results at a minimum cost. This can only be achieved if at each discrete point in time, the simulation moves forward from the current system state to the next system state choosing the largest time increment that maintains stability and acceptable accuracy. Time step management thus becomes the automatic determination or recognition by the computer program of

1. The initial time-step
2. System states requiring time-step reduction
3. The minimum allowable time-step
4. System states allowing time-step increase
5. The maximum allowable time-step.

The literature offers very little guidance in these matters. Many users of general purpose numerical integration packages seem to choose the integration parameters rather arbitrarily on a "try some numbers and see what happens" basis. Realistically, criteria for each decision must be based on the characteristics of the system being simulated. The considerations involved in establishing time-step management criteria for the ride dynamics model will be discussed here, with the bulk of the results of the investigation into this area being presented in Chapter V.

Initial time-step

One might think that the choice of the initial time-step would be rather arbitrary if the integration procedure had automatic interval adjustment built in. However, interval adjustment procedures do not automatically choose "the" correct time-step at a given point; they simply determine if a change in interval size is justified based on system responses at each discrete system state. A given system state may not yield a time-step increase even though a larger interval

chosen at the same point would be acceptable and not produce a time-step decrease. That is to say, that at a given discrete point, different time intervals might produce results that are within the bounds that yield no interval adjustment. Thus, the time-step used at each discrete point in the simulation can be influenced by the initial interval chosen. To achieve maximum average time-step, the initial time-step should be set to the maximum allowable value, allowing the adjustment criteria to force the step size smaller as required.

Time-step reduction

Time-step reduction is handled by monitoring system conditions at each point in time and, if conditions require it, causing the simulation to retreat to the previous system state, halving the time-step before advancing further. For this vehicle model, there are essentially three types of conditions that can indicate the requirement for time-step reduction.

1. The advance of the system from the previous discrete point produces one or more deflections or velocities that exceed the limits of the tables describing the characteristics of the vehicle components, e.g., the maximum possible deflection of a spring has been exceeded.
2. The advance of the system produces an impossible or unrealistic geometric configuration, e.g., the center of a wheel is below the riding surface or the angular displacement of the vehicle body is beyond stable limits.
3. Integration errors reported by the Merson formula are beyond prescribed limits.

Either of the first two conditions might be considered justification for termination of the simulation on the grounds that the conditions might be produced by invalid modeling techniques or incorrect vehicle parameters. (The original FMC model terminated upon encountering the first condition and provided no checks

for the second.) However, these system states can also be encountered simply because the integration step taken is too large, i.e., the simulation has taken a step forward too big for the conditions encountered in that step (possibly severe terrain irregularities), thus not allowing the model to respond to the external stimuli in increments small enough to simulate actual vehicle response.

In using the integration error reported by the Merson formula, normal procedure is to compare the reported error with some preselected maximum. If the reported error exceeds the maximum allowable, the interval is reduced as stated previously. Since the reported error is ideally of the order h^5, halving the time-step theoretically reduces the truncation error by a factor of 2^5 or 32.

Applying Merson's error formula to the vehicle model is not a straightforward task. Simulation of a two-axle vehicle (seven degrees of freedom) requires the integration of fourteen first-order equations, thus the possible requirement of monitoring fourteen different computed error values for each time-step. Motions of the vehicle components are of widely varying magnitudes, frequencies, and velocities. Also the presence of discontinuities inherent in the representation of the vehicle suspension parameters, the terrain profile, and the tire produce severe changes in the vehicle characteristics (in essence, changes in the system being modeled) during one time interval, especially with larger intervals. Hence, there are several questions regarding the applicability of Merson's error formula to this model.

1. Does error computed by the Merson formula accurately reflect degradation of the vehicle responses due to excessive time-step size?
2. Which integration error should be monitored?
3. Should the error be evaluated on an absolute or a relative basis, i.e., should the magnitude of the variable being computed be considered?
4. What constitutes an error threshold, i.e., that error value that requires a time-step reduction to maintain satisfactory model performance?

These questions will be discussed in Chapter V.

Minimum allowable time-step

A minimum allowable time-step is normally specified to prevent the time-step from being reduced to the point where roundoff error begins to dominate the solution, resulting in numerical instability. Roundoff error in numerical integration can be minimized if the time-step is chosen such that the derivative multiplier (h/3 for Merson's formulae) can be represented exactly by the computer using a minimum number of binary digits, preferably one. Using Merson's method this means choosing h such that $h = 3(2^n)$ where n is an integer. The resulting multiplier is 2^n which in computer real number storage requires only one significant binary digit. Always changing the time-step by a factor of two (halving or doubling) maintains this condition.

Considering the computer used in this study (Honeywell G440) the number of significant digits that can be represented is such that for the relative magnitude of derivatives and time intervals involved, roundoff is of little consequence. Thus, the minimum time-step can be based on considerations other than roundoff. Realistically, it

can be based on minimum forward motion of the vehicle in one time-step. It can be assumed that once the time-step has been reduced to the point that forward motion of the vehicle is less than some very small distance (quantified in Chapter V), further reduction of the time-step will not improve the result. If further reduction is indicated for any of the reasons previously specified, the simulation should be terminated and the cause of the malfunction investigated.

Time-step increase

Assuming that the simulation began using the maximum allowable time-step and at some point the interval was reduced for one of the three reasons specified, then at some later point (possibly after a severe disturbance has been passed), the time step could be safely increased. The only indication available as to the feasibility of increasing the interval size is the integration error computed by Merson's formula. Again, essentially the same questions arise as previously encountered with the time-step increase

1. Do very small errors justify a time-step increase?
2. Which integration error should be used as a basis for an increase?
3. Should the error be evaluated on an absolute or relative basis?
4. What constitutes the lower error threshold?

It could be argued that, ideally, for a truncation error of the order h^5, establishment of the maximum error allowed, E_{max}, would also establish the lower error threshold, E_{min}, at $E_{min} = E_{max}/32$. However, this would produce an oscillating situation in which the step size was constantly being alternately increased and reduced. A more effective value might be $E_{min} = E_{max}/50$. Realistically, the computed error

term is only a rough approximation of the error involved and it remains to be determined how it can be applied to the vehicle simulation.

Maximum allowable time-step

It would be unrealistic to allow the time-step to grow without establishing an upper bound. If allowed to grow without limit, it is conceivable that during traversal of a relatively smooth riding surface on which very little vibratory motion is occurring, the time-step could grow to the point at which forward motion in one step is such that significant terrain irregularities or obstacles are completely "jumped over" without being "felt" at all by the model. It is also well to consider natural frequencies of vehicle components when establishing a maximum time-step. One might argue that it is necessary to execute a minimum of eight or ten integrations per period of the highest frequency motion being modeled. Thus the maximum time-step might be established as a function of the forward velocity of the vehicle or of the period of vibration of some vehicle component. Criteria will be established for this in Chapter V.

Description of the Computer Program

The computer program (Appendix) is written in FORTRAN IV for a Honeywell G440. Conversion to any other digital computer with a FORTRAN capability should be relatively simple to accomplish. The only problem that might be encountered in such a conversion is the introduction of significant roundoff error in some machines with less than ten

significant decimal digits in its representation of floating point numbers.

The program begins by reading user-provided parameters describing the vehicle. These include the weight of the body, the body moment of inertia in pitch and roll, the position of each axle relative to the body center of gravity, the weight and roll moment of inertia of each axle assembly, the position of each suspension and each tire, and the spring and damper characteristics of each suspension and tire. The springs and shock absorbers are described by force-deflection and force-rate tables. The tires are described by specifying the number of segments to be used, the angle to each segment center line, and the spring constant for each segment. The tire damping characteristics are read as force-rate tables, respectively. In preparing force-deflection tables for the suspension springs, the effect of limited relative displacement between the axle and body, i.e., the axle assembly hitting the bump stop, should be provided for by a very stiff force-deflection relationship beyond that point of maximum spring deflection.

The program next reads the riding surface profile which is described simply as (x,y) coordinates relative to the initial position of the vehicle. A different profile can be entered for each side of the vehicle so that terrain can be depicted as actually encountered, which may be considerably different for the right and left tires.

If the static equilibrium configuration for the vehicle is known (the deflection of each tire and suspension spring at static equilibrium), this can be read in and the program can proceed directly to the simulation of forward motion. If the static equilibrium

configuration for the vehicle is not known or if it is desired to simulate a vertical drop of the vehicle (with no forward motion), then an arbitrary initial configuration can be read in and the program will provide simulation of vehicle oscillation until all motion is damped out and static equilibrium is reached. This is a rather time-consuming procedure and should be avoided once the static equilibrium configuration has been recorded for a given set of vehicle parameters.

Having been given the static equilibrium configuration, the program reads the initial forward velocity and the forward acceleration desired. (This latter must be constant and is usually zero.) Next, reading the stop distance or the stop time completes the definition of the simulation required.

Then, simulation of vehicle responses to forward motion over the specified riding surface begins and continues until stop distance or stop time is reached. The procedure essentially consists of determining the forces on the vehicle masses produced by the current displacements and velocities (system state), relating forces and masses to determine accelerations, and integrating accelerations successively to determine velocities and displacements at the next point in time. This new system state produces new forces and the procedure begins anew. The time step is adjusted if required as described previously. All system responses at each point in time are recorded on permanent storage devices so that retrieval for generating graphical displays or for statistical analyses is possible.

CHAPTER IV: VALIDATION FOR A REPRESENTATIVE 4 X 4 VEHICLE

To establish the validity of the model, it was decided to simulate a military vehicle for which measured vehicle responses were available from field tests previously conducted by the MRB of the WES, Vicksburg, Mississippi. The vehicle chosen was the M37--a 3/4-ton, four-wheel drive truck used to transport personnel or light cargo. In the field tests the truck was instrumented with accelerometers to record vertical accelerations at three points--on the front axle near the right wheel, at the body center of gravity, and on the truck body under the driver's seat. It was also equipped with a gyro to record the pitch motion of the vehicle.

Vehicle Parameters

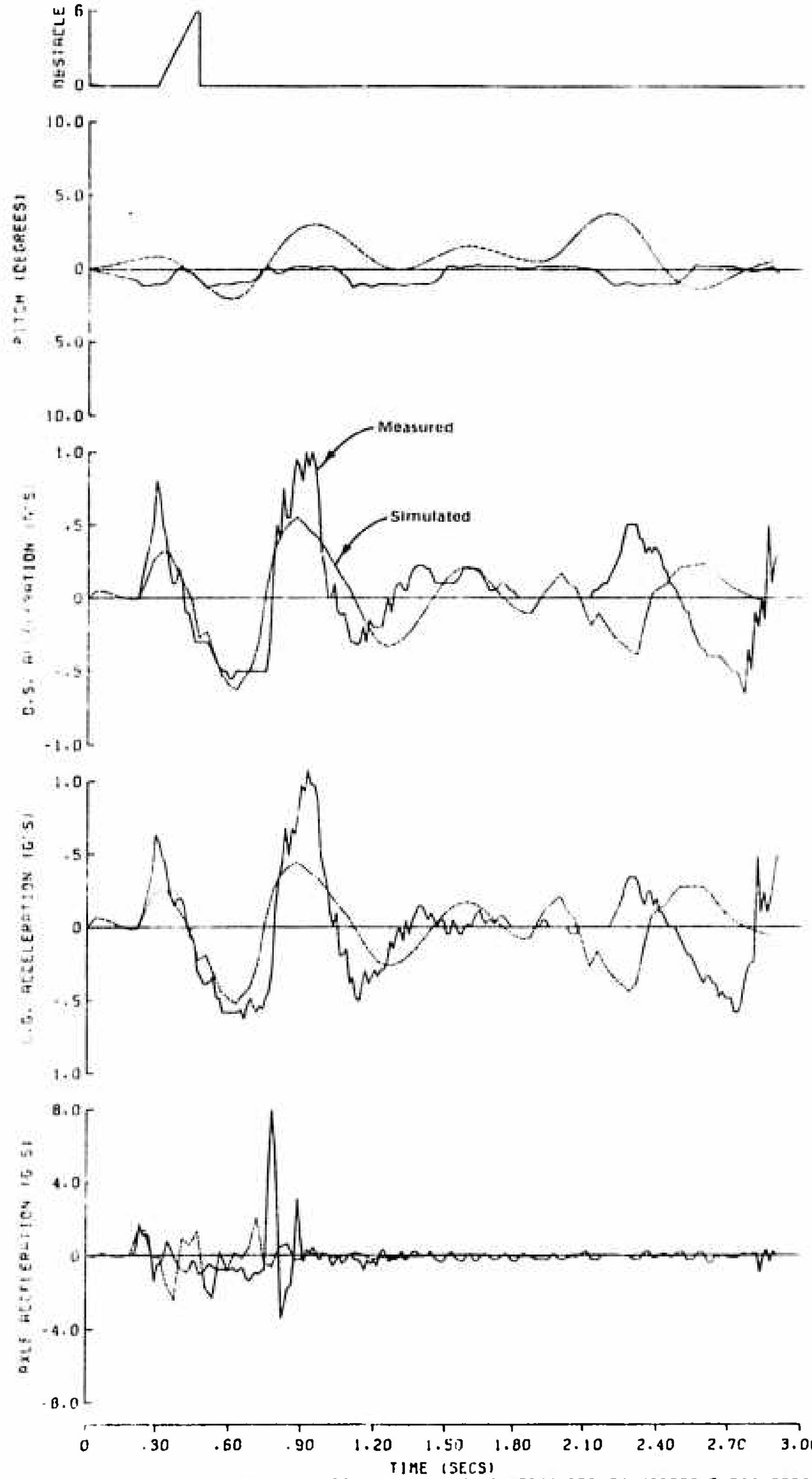
The vehicle parameters used in the simulation, with the exception of the tire data, were taken from information provided to WES by the FMC Corporation. It is known that the force-deflection relationships for the springs and the force-rate relationships for the shock absorbers were measured from disassembled vehicle components and that the parameters describing the relative geometric position of the vehicle axles, suspensions, tires, etc., were measured. It is thought that the moment of inertia values are computed values and are of questionable accuracy. The tire segment spring stiffness was computed as described in Chapter II using measurements made by the MRB. Tire damping was computed by the MRB based on drop tests conducted with a single wheel. The tire parameters are considered reliable.

Courses Traversed

Four field tests were chosen for simulation--three involving single obstacle traversal only and one cross-country test. The obstacle tests were for a single obstacle size and shape--a six-inch-high, thirty-degree ramp-type wooden obstacle. The obstacle data used represented obstacle impact at three different speeds--5.65 ft./sec., 15.7 ft./sec., and 22.0 ft./sec. The cross-country run simulated consisted of traversal of 300 feet of hard, rocky terrain located at a WES test site in Nevada. The test was run at approximately eight miles per hour.

Predicted and Measured Responses

Figures 8, 9, and 10 show the measured and predicted vehicle responses for the three obstacle tests. In each case, the dashed lines represent the computed values and the solid lines represent the measured values. Beginning at the bottom of each page, the first curve plotted represents simulated and measured vertical acceleration of a point on the front axle near the right wheel, where an accelerometer was located during the tests. The next plot represents vertical acceleration at the center of gravity of the truck body, and the third depicts acceleration at a point on the truck body located beneath the driver's seat. The fourth plot represents computed versus measured pitch of the truck body. It should be noted here that for Test No. 157 (Figure 8), the pitch-measuring instrumentation apparently malfunctioned during the test, resulting in no valid record of measured pitch for that



COMPARISON OF MEASURED AND SIMULATED VEHICLE RESPONSES TO OBSTACLE TRAVERSAL
FORWARD VELOCITY OF VEHICLE = 5.65 FT/SEC : TEST NO. 157

Figure 8. Simulated Versus Measured Responses - Test No. 157

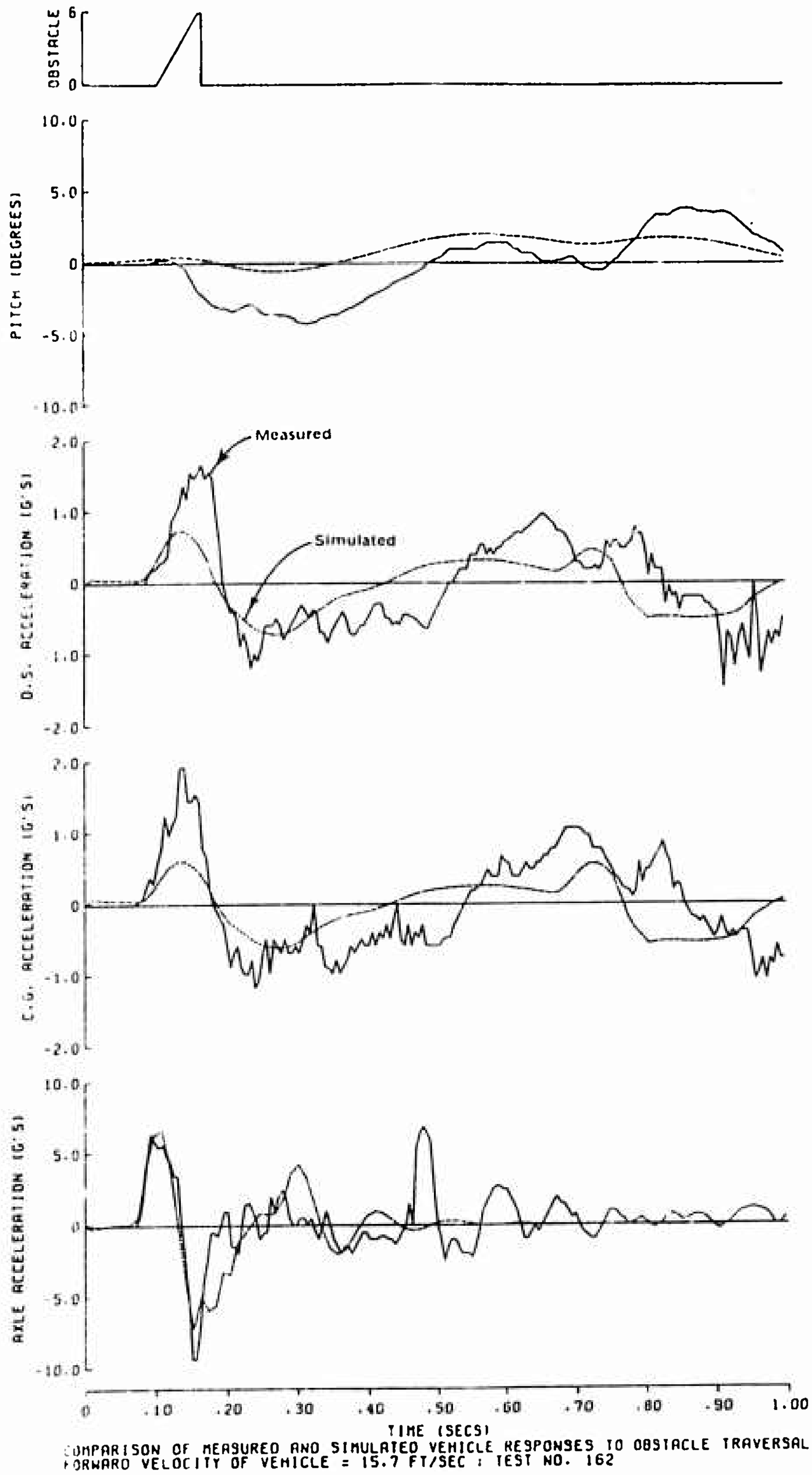


Figure 9. Simulated Versus Measured Responses - Test No. 162

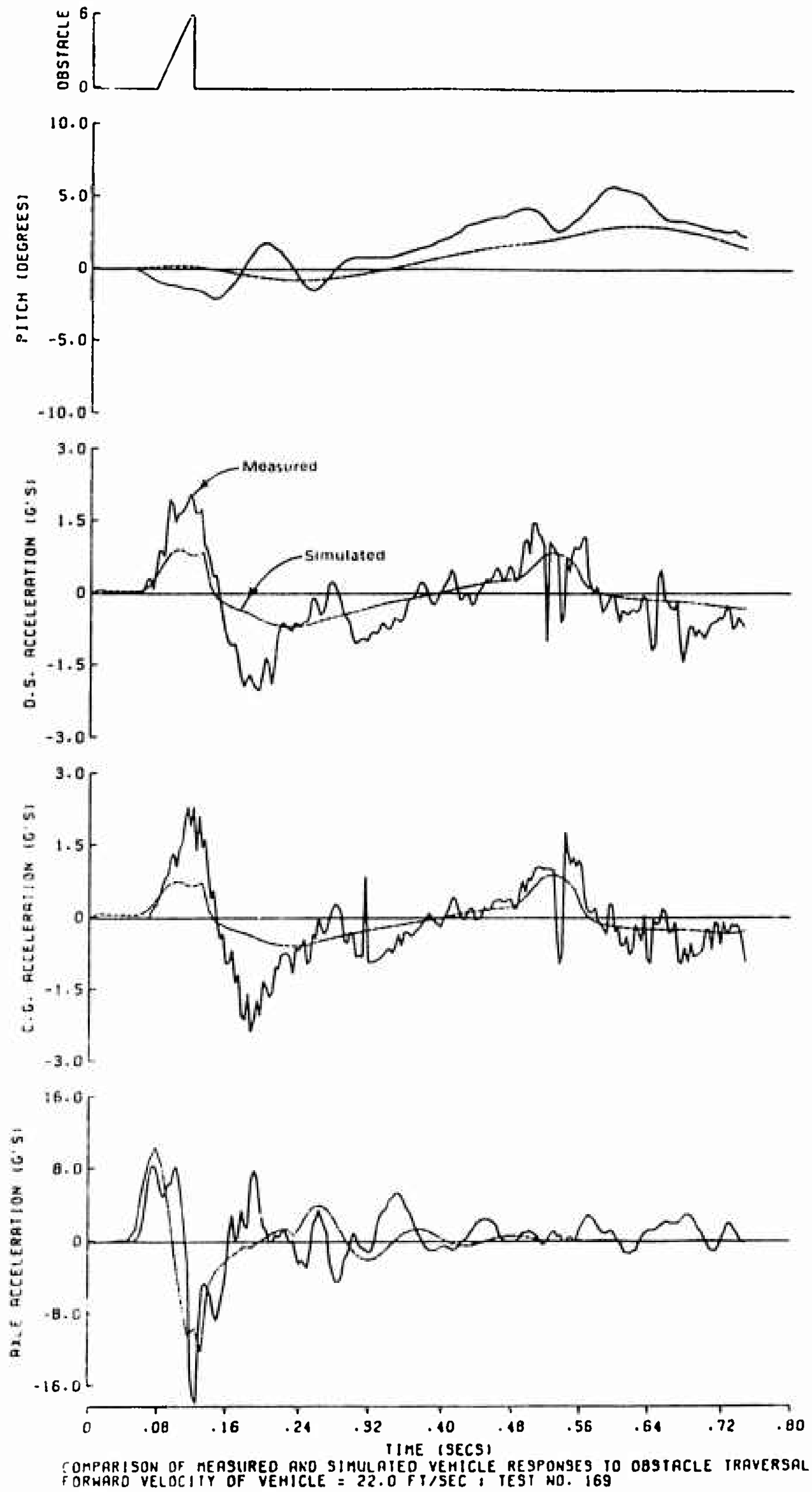


Figure 10. Simulated Versus Measured Responses - Test No. 169

test. At the top of each figure, the position of the obstacle relative to the front axle of the vehicle is shown, i.e., the obstacle as shown represents the displacement function for the front tires plotted on the same time scale as the vehicle responses.

In displaying measured versus predicted data for the cross-country run, the duration of the run, the uncertainty of the exact speed of the vehicle at any given point, and the possibility of yielding or inaccurately defined terrain at points make it impractical to display actual responses such as those for the obstacle tests. Also the root mean square (RMS) of vertical accelerations felt by the driver throughout the duration of the test is considered to be of greater significance than isolated acceleration peaks. Thus, the cumulative RMS of predicted versus measured vertical acceleration under the driver's seat for 300 feet of the cross-country test is plotted as Figure 11. As with the previous figures, the dashed line represents the simulated response.

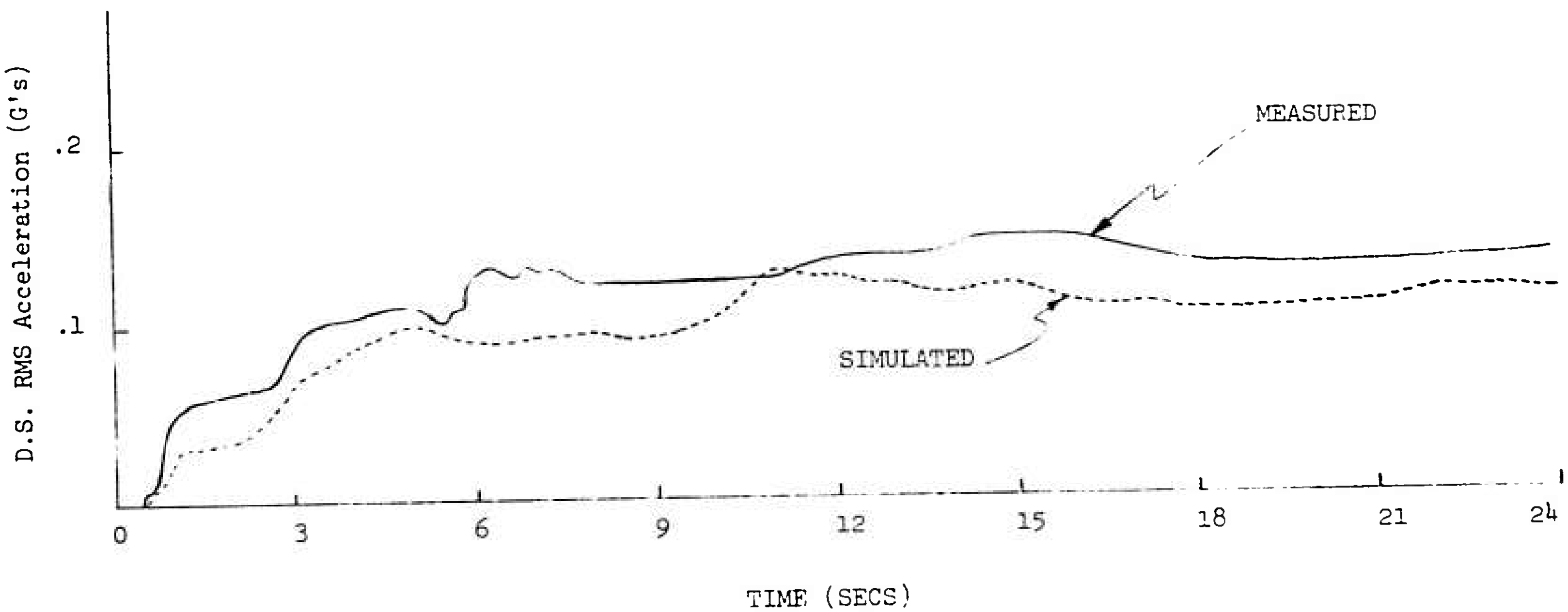


Figure 11. Simulated Versus Measured RMS Acceleration Under Driver's Seat for Cross-Country Run

CHAPTER V: DISCUSSION

Comparison of Model Responses Against the Field Data

In comparing the predicted and measured accelerations for the obstacle tests (Figures 8, 9, and 10), it should first be noted that the high-frequency oscillations present in the measured data are due in part to high-frequency motions such as frame flexure and engine vibration which are not included in the mathematical model.

Looking first at acceleration of the front axle, the positive acceleration produced by the intial impact with the obstacle and the negative acceleration from the rebound that follows, i.e. the first complete cycle of axle motion, compare favorably in shape and amplitude. This is especially true of the higher speed tests in which the amplitudes of the acceleration in the first cycle are greater. Beyond the first cycle, the measured and predicted acceleration curves are displaced in phase, and there is much greater measured motion than simulated motion. The axle motion produced by the model seems to have been damped out whereas the measured data continue to show oscillations. Generally, the predicted axle accelerations which represent the most sensitive vehicle response that can be monitored are better than expected.

The center of gravity (CG) and under driver's seat (DS) acceleration curves are very similar in all cases--the driver's seat is located very close to the center of gravity--and will be discussed together. The general shape of the predicted curves is good, particularly during the first cycle, but there is obviously a problem with amplitude. The

predicted amplitude is generally too low throughout the tests, particularly for the positive accelerations on initial impact. The amplitudes of the negative accelerations on rebound from obstacle impact are much closer to the measured data, especially for Tests No. 157 and 162 (Figures 8 and 9, respectively). As with the axle accelerations, there seems to be some phase difference beyond the first cycle. Table 1 shows a comparison of the simulated versus measured driver's seat RMS accelerations at the end of the three tests. While there is not close agreement in the final RMS values, the differences which average 29 percent for the three tests are not as drastic as the raw acceleration curves might imply.

In evaluating the pitch curves (ignoring Figure 8), it is difficult to defend the simulated pitch motion. The overall trend of the pitch curve for Test No. 169 shows some resemblance to the measured values while the curves for Test No. 162 bear little resemblance. However, there must be some question as to the validity of the measured pitch motion. The measured pitch curves for Tests 162 and 169 should be very similar in shape and amplitude with a large initial negative pitch and a positive pitch toward the end of the tests as the rear wheels strike the obstacle. There are considerable differences in the two measured pitch curves, enough to cast doubt on one or both curves. Still, it must be admitted that the model has apparently severely under-predicted the pitch motion.

Figure 11 shows the measured versus simulated RMS acceleration under the driver's seat for the cross-country run. The test was run at approximately eight miles per hour over rough terrain. The degree of agreement between the measured and simulated curve is considered to

Table 1

Measured and Simulated RMS Accelerations for Obstacle Tests

Test No.	Speed (ft./sec.)	RMS Acceleration (G's)	
		Measured	Simulated
157	5.65	0.35	0.25
162	15.7	0.51	0.37
169	22.0	0.61	0.42

be within acceptable limits. The average difference between the two curves is approximately 15 percent. While this is encouraging, the model is predicting less than measured values and the model should ideally predict greater accelerations for this type of test. The simulation deals with a nonyielding terrain, whereas there was probably at least some terrain deformation which reduced the measured vehicle accelerations from those that would be encountered traversing a completely rigid riding surface. Also the straight-line representation of the terrain used in the simulation produces abrupt changes in tire deformations that may be somewhat harsher than those produced by actual terrain.

In trying to determine reasons for the lack of agreement between the measured and simulated vehicle responses, particularly the obstacle tests, there are basically two sources of error that could result in bad comparisons for an essentially valid model.

First, the field test results are not repeatable. The forward velocity of the vehicle was controlled by the driver using the accelerator pedal. While he attempted to maintain constant speeds throughout the tests, the speed actually varied considerably as the truck ran over the obstacle. The simulation maintains a constant speed absolutely. This somewhat explains the apparent phase difference between the measured and computed accelerations beyond the first cycle. Also the steering is controlled by the driver; thus, the angle at which the vehicle strikes the obstacle is subject to human error. Duplicate field tests run over the same obstacle at the same speed produce measured acceleration maximums that are commonly different by fifteen to twenty percent.

Secondly, with the exception of the tire data, the vehicle parameters used in the simulation are questionable. The suspension parameters provided by FMC were measured from components disassembled from a specific vehicle, not the same vehicle used in the WES field tests. MRB has learned that characteristics of springs and shock absorbers can vary significantly from unit to unit of the same type vehicle. Also, the vehicle moments of inertia were apparently computed by FMC for a vehicle loaded with a considerably different weight distribution than the vehicle used in the WES tests. For the WES obstacle tests, the load was arranged to produce equal weight on all four wheels. Judging from the pitch motion curves, the pitch moment of inertia used in the simulations was much higher than the actual moment of inertia for the vehicle as loaded for the obstacle tests.

While the figures presented represent the extent of the validation effort possible in the time frame available, it is felt that much closer comparisons between predicted and measured data could be obtained by altering the vehicle parameters, i.e. "tuning" the model to match the measured accelerations. However, this would be a time-consuming process and would not necessarily be a valid approach to the problem. Ideally, complete validation should be accomplished using vehicle parameters obtained through tests and measurements on the field test vehicle, assuring parameter reliability.

Investigation of Effects of Integration Step Size

In an attempt to establish criteria for control of the integration step size for the model, simulation of obstacle traversal was executed using several different constant time intervals. First, using a forward velocity corresponding to the highest speed validation test (No. 169), a "reference" time-step was established by executing the complete simulation at a constant time-step, reducing the time-step by half and executing again. This was repeated until further reduction in the time-step produced no detectable change in the axle acceleration curve, axle acceleration representing the most sensitive vehicle response. Execution of the simulation at this reference time-step represents the "best" simulation results possible from the model.

Having established the reference time-step (0.00195 sec. in this case), plots were made to compare vehicle responses produced by the reference simulation with responses produced by simulations executed at time-steps of (2^n) T, where T is the reference time-step and n is a positive integer. Figures 12 through 15 show the results of this procedure. The responses chosen for comparison were front-axle acceleration, velocity, and displacement, and acceleration under the driver's seat. Axle acceleration represents the response in which effects of changes in time-step are most obvious; axle velocity and displacement are, in effect, the forcing functions to the vehicle body; and the driver's seat acceleration is actually the vehicle response of primary concern. In each plot, the solid lines reflect the reference time-step responses and the dashed lines the simulation at the larger time-step. Also plotted with the axle velocity and displacement are the

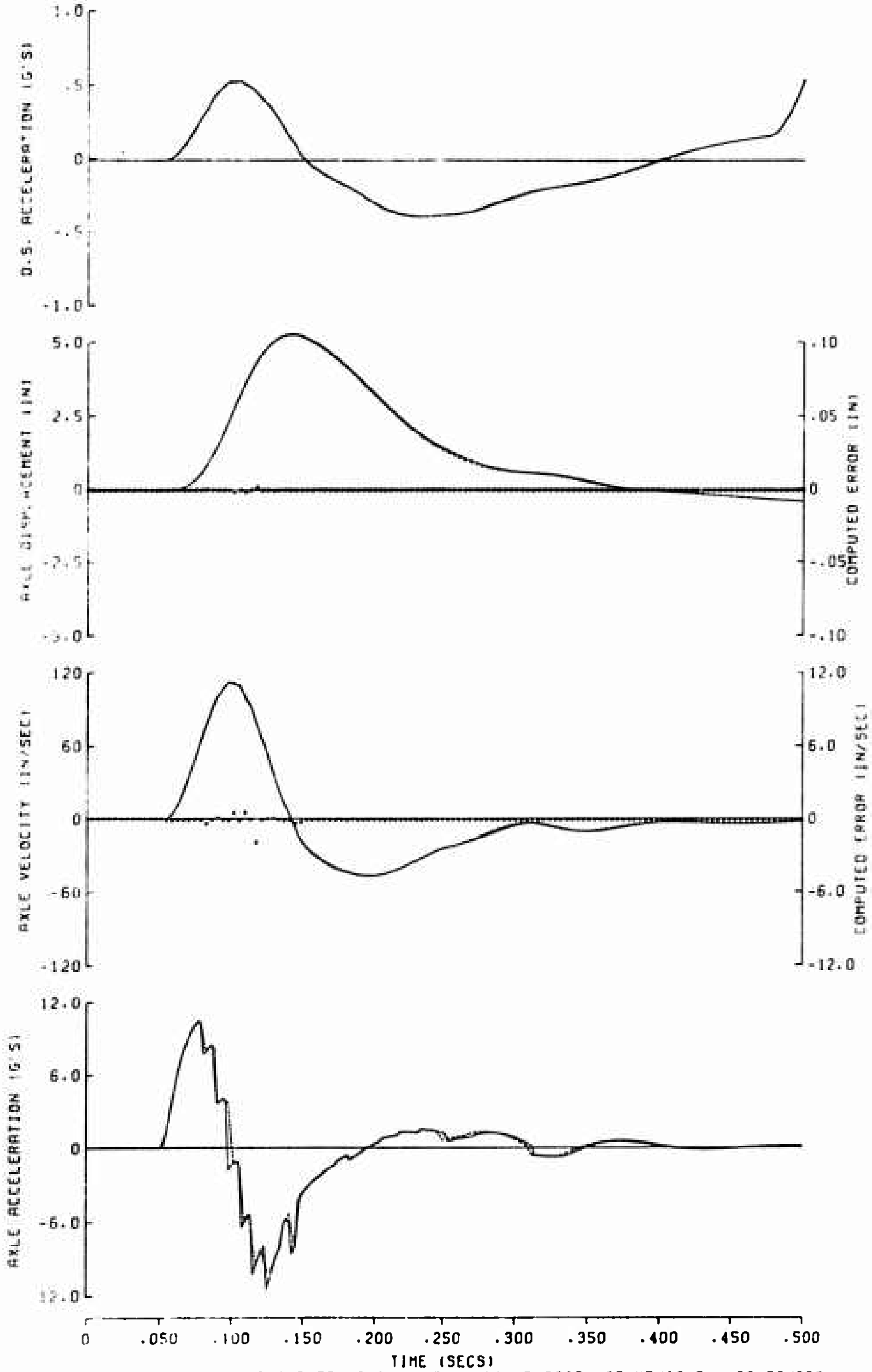


Figure 12. Comparison of 22-fps Responses Using 2 Times Ref. Time-Step

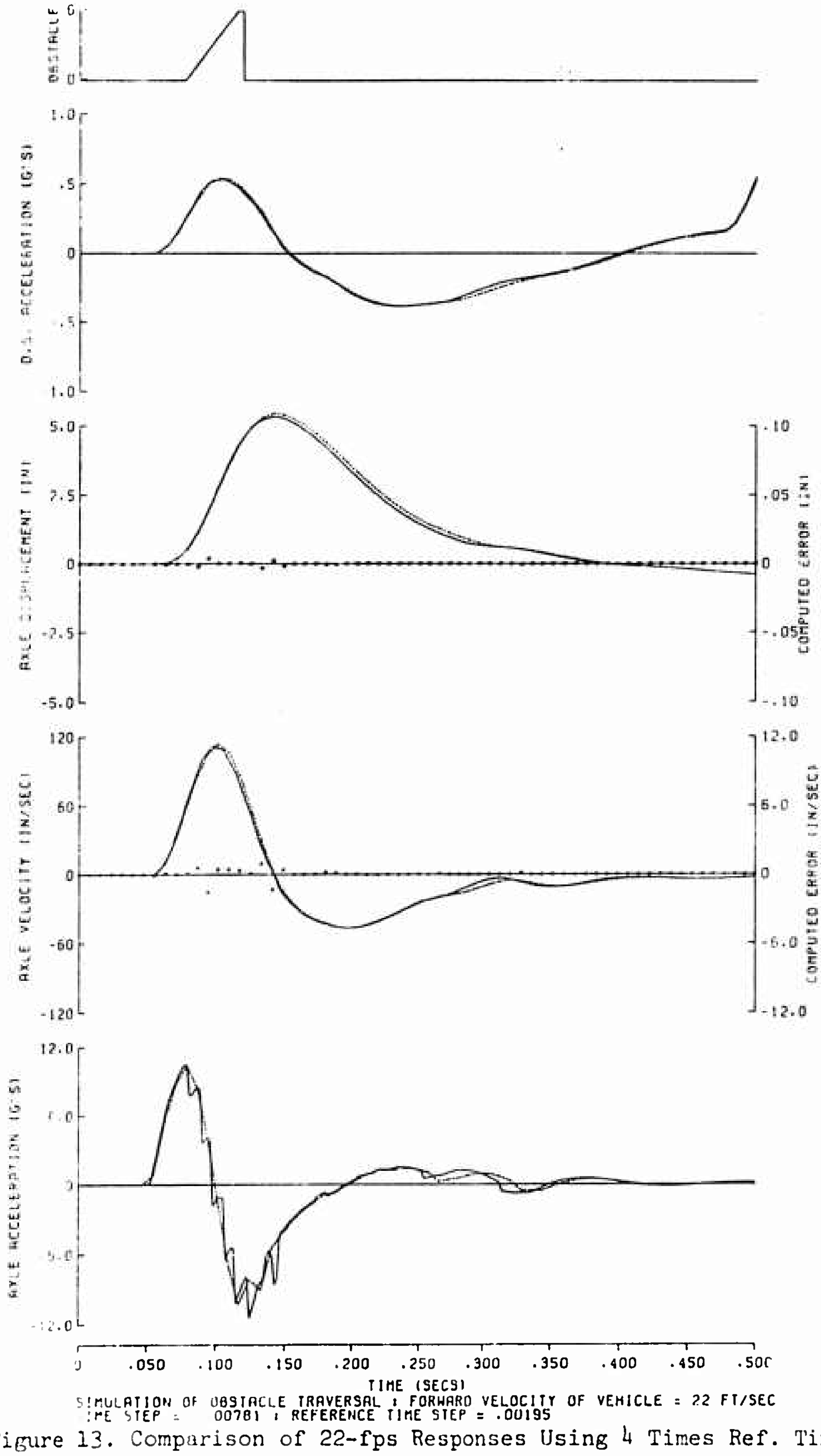


Figure 13. Comparison of 22-fps Responses Using 4 Times Ref. Time-Step

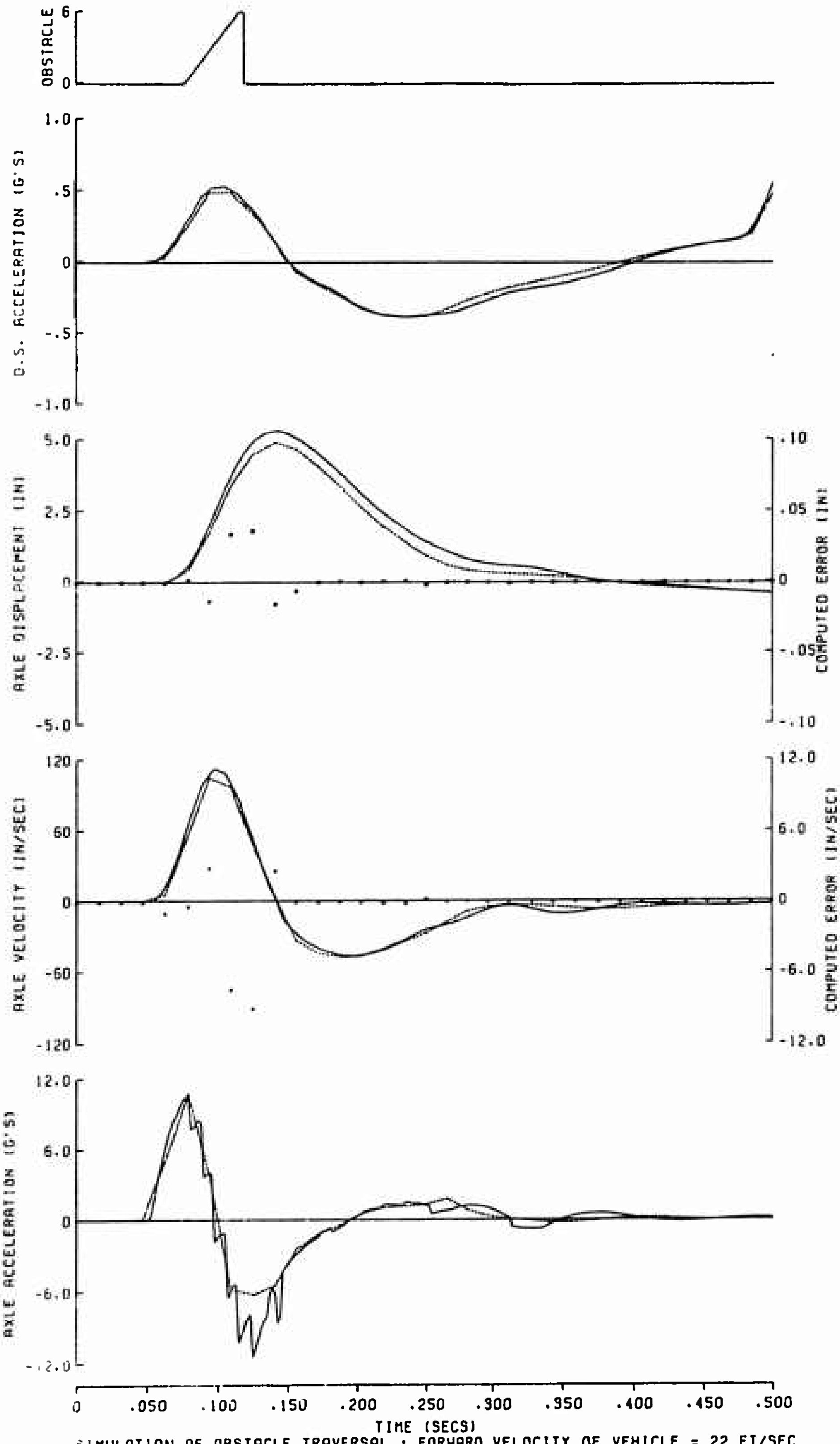


Figure 14. Comparison of 22-fps Responses Using 8 Times Ref. Time-Step

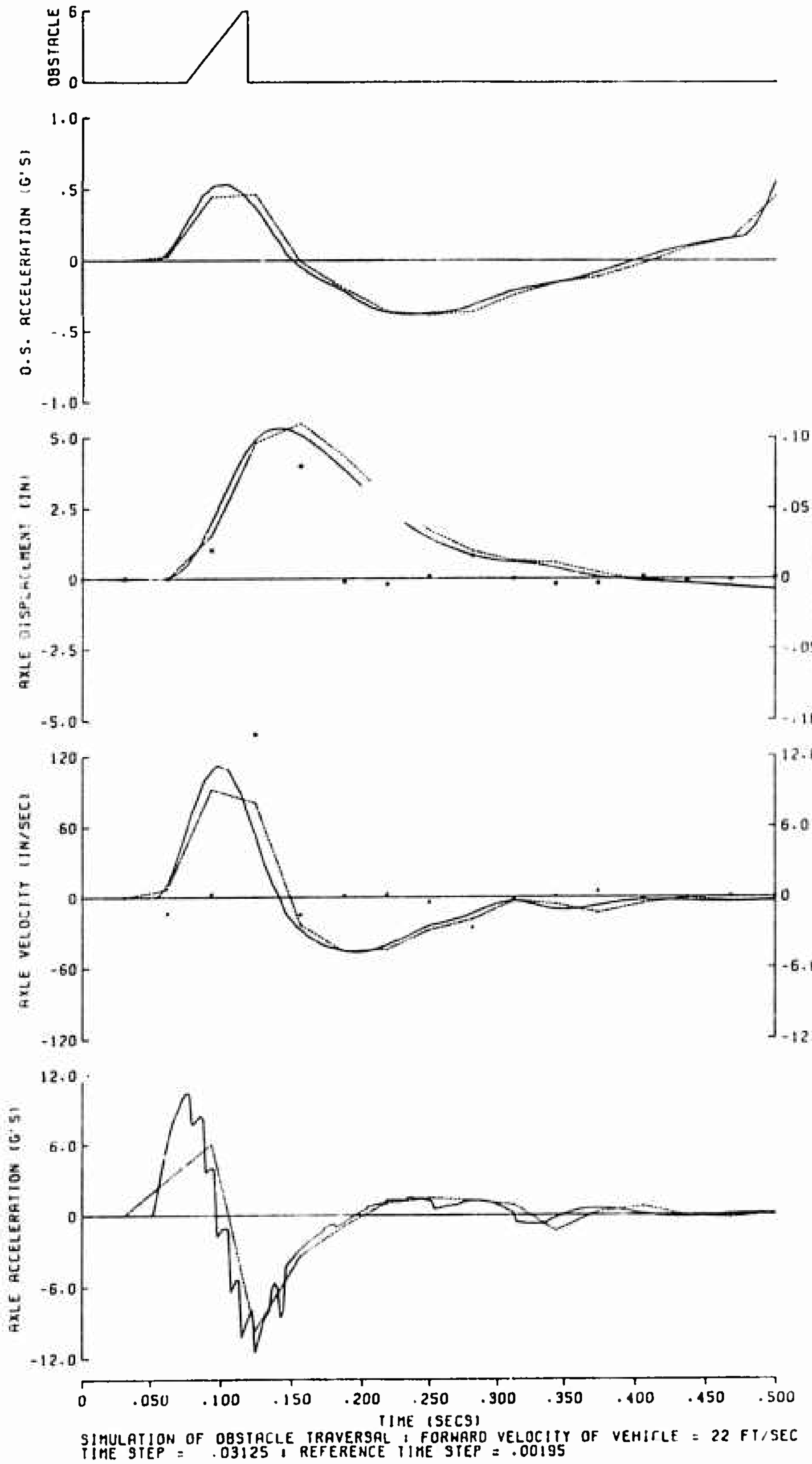


Figure 15. Comparison of 22-fps Responses Using 16 Times Ref. Time-Step

error terms, computed by Merson's error formula, for these two integrations. These are plotted as discrete points, the scales for these points being drawn on the right side of the figure.

Beginning with Figure 12, the dashed lines represent simulation at double the reference time-step. While the axle acceleration shows some departure from the reference, all the other curves are essentially the same. Obviously, this time-step would be acceptable throughout the simulation. Figure 13 reflects use of a time-step of four times the reference. Here the axle acceleration shows a further departure from the reference, and the velocity and displacement curves are beginning to show a slight deviation. However, the driver's seat acceleration is still essentially unchanged. Looking at Figure 14, which represents a time-step of eight times the reference, the axle acceleration has departed significantly from the reference curve, the velocity and displacement show some error, but the driver's seat acceleration curve is still very close to the reference curve. Doubling the time-step again (now sixteen times the reference) yields Figure 15. Here the axle acceleration is severely degraded and the velocity, displacement, and driver's seat acceleration curves show a further departure from the reference than did curves produced by smaller time-steps, but the critical driver's seat acceleration still shows very good shape and amplitude and is still, for all practical purposes, acceptable for computing RMS acceleration. Attempting to increase the step-size to 32T produced a condition in which the maximum value of one of the suspension spring force-deflection tables was exceeded (a condition described in Chapter III as indicating need for time-step reduction),

thus making further simulations at larger constant time intervals inpracticable for this vehicle speed.

Not being able to judge as unacceptable the results from any of the time steps used for the 22 ft./sec. simulation, it was decided to repeat the procedure above, simulating obstacle traversal at twice the forward velocity, i.e. 44 ft./sec. The reference time-step was established as 0.00098 seconds using the same procedure as previously described. Figure 16 shows the results of simulation at twice the reference, obviously revealing no significant deviation in any of the responses. Figures 17, 18, and 19 reflect the responses computed using time-steps of 4T, 8T, and 16T, respectively. These reveal essentially the same pattern as would be expected based on the previous series of figures, i.e., some departure from the reference axle accelerations, velocities, and displacement, but little significant difference in the driver's seat acceleration curves. However, Figure 20, which represents execution using an integration time-step of 32T, deserves further comment. Here the axle acceleration is severely degraded bearing little resemblance to the reference curve. With this type of axle acceleration curve, one would expect that the vehicle responses in general, including driver's seat acceleration, would show a significant departure from the reference curves. This is not the case here. Axle velocity and displacement and consequently the driver's seat acceleration still show the essential effects of traversal of the obstacle. For the driver's seat acceleration, the positive amplitude, the time of the positive peak, the negative amplitude, and the time of the negative peak are all close to the reference

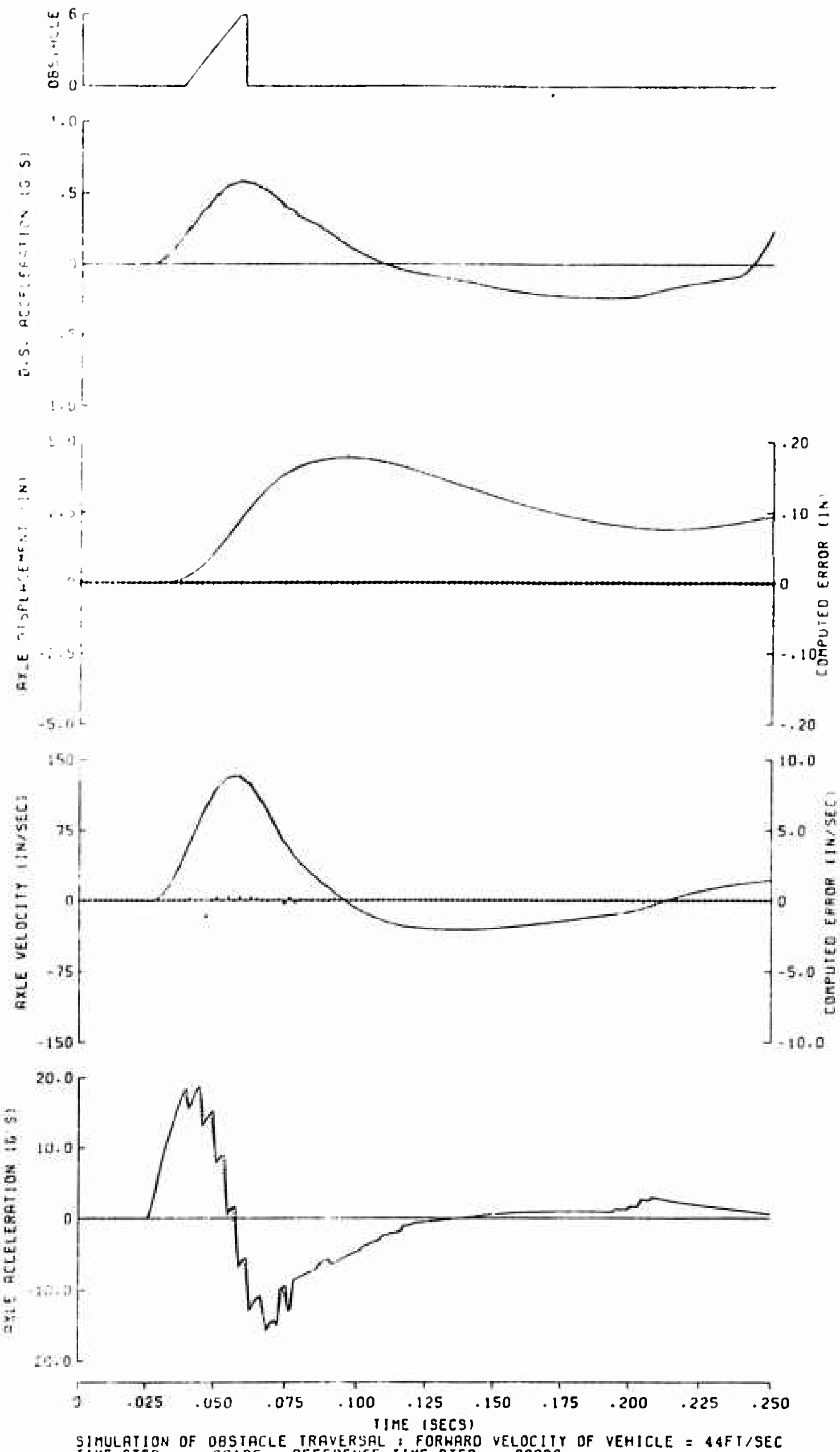


Figure 16. Comparison of 44-fps Responses Using 2 Times Ref. Time-Step

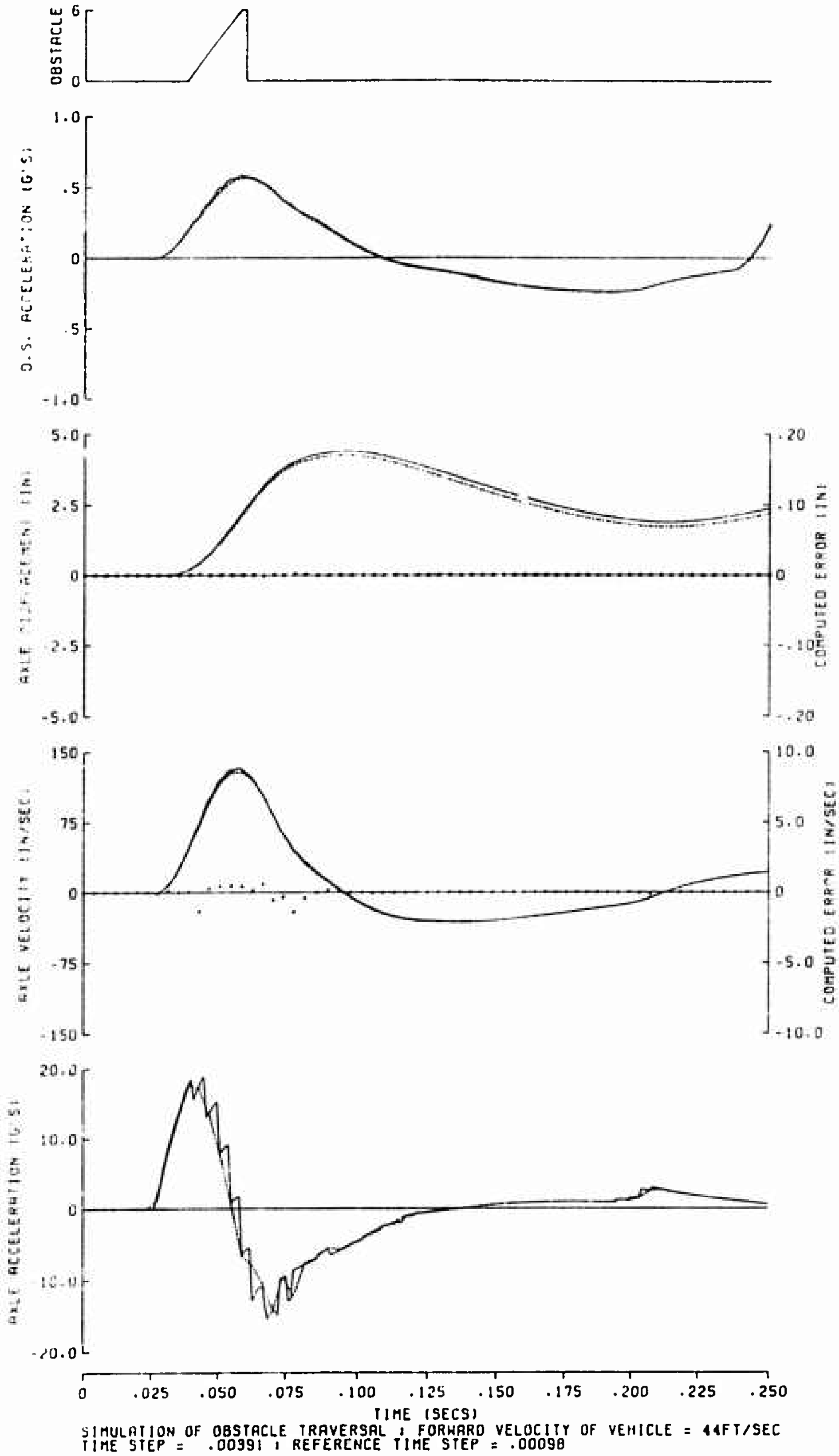


Figure 17. Comparison of 44-fps Responses Using 4 Times Ref. Time-Step

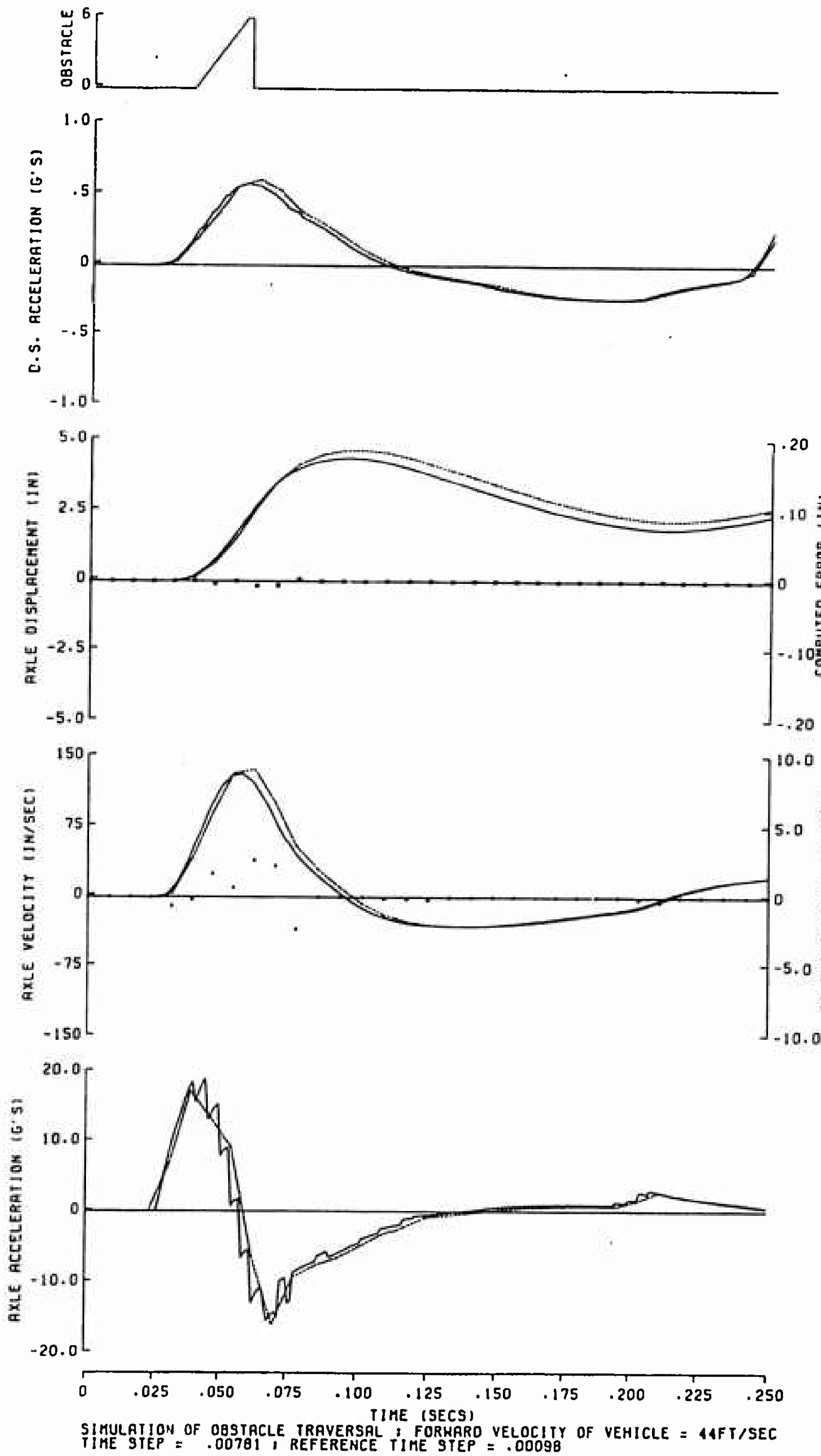


Figure 18. Comparison of 44-fps Responses Using 8 Times Ref. Time-Step

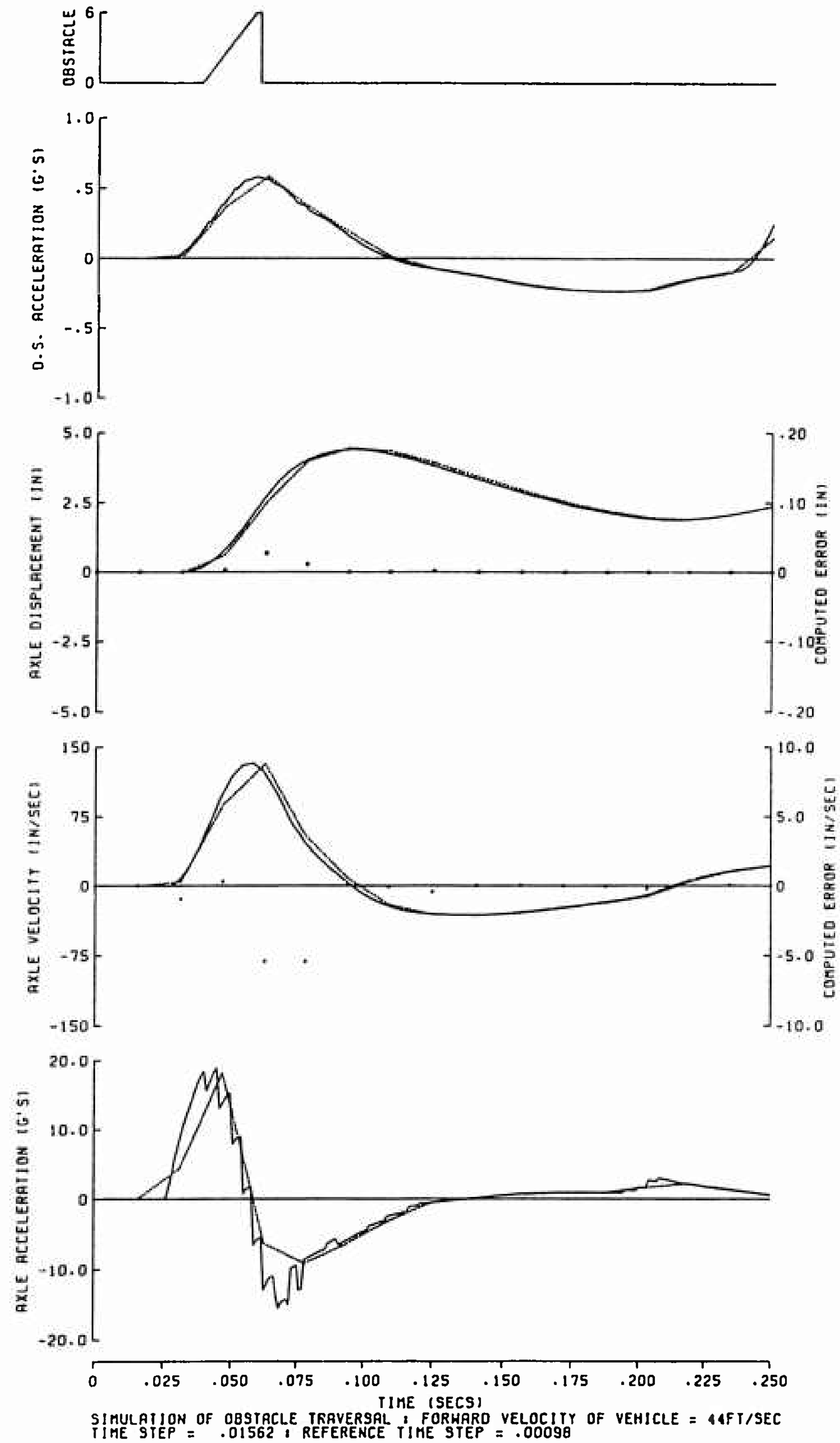


Figure 19. Comparison of 44-fps Responses Using 16 Times Ref. Time-Step

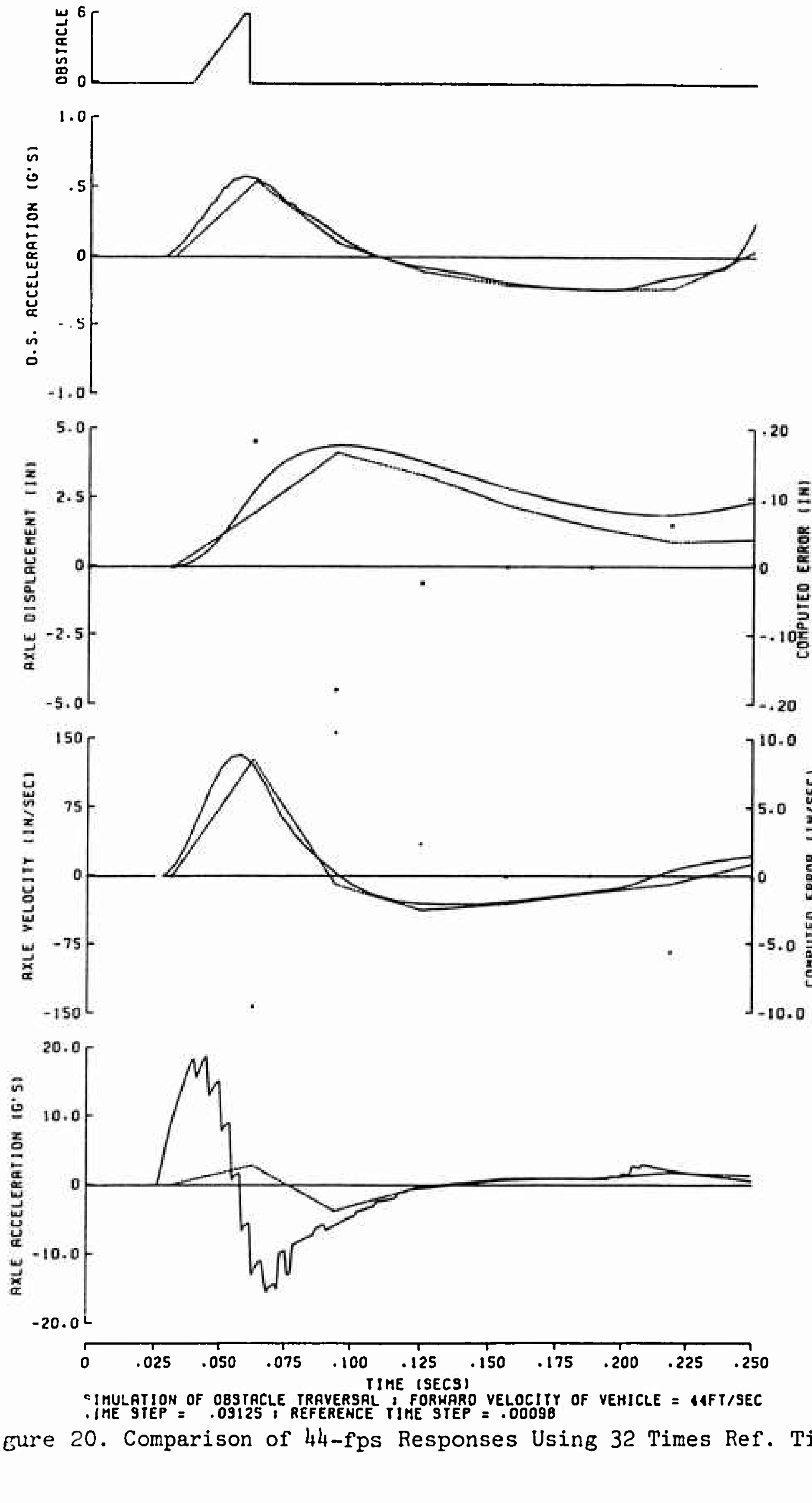


Figure 20. Comparison of 44-fps Responses Using 32 Times Ref. Time-Step

values. Attempting to execute the simulation at a time-step of 64 T produced the condition described earlier in which a vehicle parameter table was exceeded. Again, it was not possible, within the realistic limits of suspension components, to execute simulation at a time-step large enough to severely degrade the vertical acceleration at the significant point on the vehicle body, i.e. the driver's seat.

To aid in extracting conclusions from the figures, Table 2 represents a summary of the results. Column 1 specifies the forward velocity in ft./sec. (22 or 44). Column 2 is the time-step being compared with the reference. Column 3, labeled "ΔS," indicates the forward motion (inches) of the vehicle per time-step. Column 4 is the maximum error (in./sec.) computed by the Merson formula in the integration yielding axle velocity. Column 5 is the maximum error (inches) computed by the Merson formula in the integration yielding axle displacement. Column 6, labeled "RMS Acc. Residual," is the absolute difference (G's) between the compared and the reference driver's seat RMS acceleration at the end of the simulation. Column 7, labeled "Avg. Acc. Residual," represents the average absolute difference (G's) between the compared and the reference driver's seat accelerations for all discrete points in the simulation.

How can these results be related to the basic questions concerning time-step management posed in Chapter III? First, consider determination of maximum time-step. As stated previously, it is not uncommon to relate maximum time-step to the period of vibration of the highest frequency component being modeled. In this case this means vertical motion of the axle, which for the vehicle simulated

Table 2

Summary of Obstacle Traversal Simulation at Various Time-Steps

FORWARD VELOCITY (fps)	TIME STEP	ΔS	MAX. VEL. ERRORS	MAX. DISP. ERRORS	RMS ACC. RESIDUAL	AVG. ACC. RESIDUAL
22	0.00391	1.03	1.98	0.0015	0.001	0.004
22	0.00781	2.06	1.56	0.0040	0.005	0.011
22	0.01562	4.12	9.00	0.036	0.009	0.021
22	0.03125	8.24	13.9	0.11	0.015	0.031
44	0.00195	1.03	1.10	0.	0.	0.002
44	0.00391	2.06	1.38	0.0012	0.004	0.006
44	0.00781	4.12	2.65	0.0060	0.006	0.019
44	0.01562	8.24	5.35	0.028	0.003	0.016
44	0.03125	16.48	10.5	0.18	0.006	0.033

vibrates at approximately ten cycles per second. This would indicate a maximum time-step, using ten integrations per period, of about 0.01 second. However, in simulations at both 22 ft./sec. and 44 ft./sec., time-steps in excess of 0.01 second produced acceptable results. In fact, it was seen that the accuracy of the modeled axle acceleration was not critical to producing acceptable vehicle body accelerations. To confirm the apparent conclusion that the time-step need not be limited by 1/10 the axle vibration period, the cross-country run described previously was simulated at a time-step of 0.03125 second which is approximately three times this limitation. The driver's seat RMS acceleration history produced is virtually identical with that plotted on Figure 11, which was produced using a time-step of 0.00781 second. This indicates that the period of axle vibration warrants consideration when choosing maximum time-step only if the axle acceleration itself is of primary importance, which is not the case for the current study. Thus, the maximum time-step can, in general, be limited by a maximum allowable forward displacement of the vehicle in one time-step. To avoid missing significant terrain features or creating unrealistic tire deflections by too great a forward advance, the time-step should be limited such that the forward displacement of the vehicle in one time-step does not exceed the diameter of the tire body. Note that the last step size used (44 ft./sec.) exceeds this limitation and would not be used in the production computer program (in which this maximum interval criterion is now implemented.)

As stated in Chapter III, the minimum allowable time-step can be established to correspond to some minimum forward displacement,

i. e., that forward displacement corresponding to the "reference" time-step defined previously. Both reference time-steps established (0.00195 at 22 ft./sec. and 0.00098 at 44 ft./sec.) correspond to forward displacement of approximately 0.5 inch. It seems reasonable then to assume that the minimum time-step can be established as a function of forward velocity, i.e., that time-step that corresponds to forward displacement of 0.5 inch. Attempts to reduce the time-step below this value will produce termination of the simulation.

Turning next to the questions regarding time-step reductions, if driver's seat acceleration is the only time history of interest, then the computed Merson error is not as significant a factor in time-step reduction as previously thought. Operating at the maximum time-step as specified above, reducing the time-step only when the requirement is indicated by excessive deflections or velocities or by geometric considerations would probably produce acceptable results in most cases. However, it is wise to monitor the Merson error, as the other conditions mentioned are not completely reliable indicators of time-step problems, e.g., a forward step too large for current terrain conditions does not always produce a table-look-up error. In determining which integration error to monitor, it was recognized that if axle acceleration is the vehicle response most sensitive to changes in terrain conditions, it follows that integration of axle accelerations to produce velocities should provide the most responsive error indicator. It was decided that velocity errors should be evaluated on an absolute rather than a relative basis since relative error evaluation produces the potential of over-rating the error significance at points where the magnitude

of the integration variable (velocity) is small, e.g., near zero crossing. From Table 2 it appears that the computed error does somewhat reflect degradation in vehicle responses. However, the computed error is obviously not a function of h^5; doubling the time-step has not produced error terms increased by a factor of 32. In one case, doubling the time-step actually resulted in decreased maximum reported error. Moreover, the computed error does not correlate well with the RMS acceleration residual or the average acceleration residual. Thus, the determination of a maximum allowable integration error must be made on a rather arbitrary basis. From Figures 12 through 20 and the corresponding maximum velocity errors, it seems acceptable to establish a maximum allowable integration error at 10 in./sec. This, in conjunction with the other conditions that produce step size reduction, should effectively reduce the time-step when required. The role of time-step reduction assumes much less significance when a realistic maximum time-step has been established as previously defined.

Finally, considering possibilities for time-step increase, it is apparent from the figures that curves that very closely approximate the reference are accompanied by very small integration errors. Thus, increase in time-step based on computed integration error is apparently a valid procedure for this model. Again, as with the maximum error, the minimum error selection is an arbitrary decision since there is no well-defined relationship between integration error and degradation of vehicle response. However, an arbitrary choice will accomplish the objective which is to increase the step-size back to the maximum as soon as possible after a severe disturbance

(such as the obstacle used here) has been passed. Based on Figures 12 through 20, an acceptable minimum error can be established at 1 in./sec. A computed axle velocity error less than 1 in./sec. will result in the step size being doubled for the next integration step.

It is interesting to note the small peaks present on the axle acceleration curves, produced using small time intervals, particularly those produced by the reference curves. These small abrupt changes in axle acceleration can be attributed to discontinuities in tire stiffness inherent in the segmentation scheme described previously. Each small peak can be related to a segment making or breaking contact with the obstacle as the tire moves forward. While undesirable effects of segmentation are apparent when viewing the axle acceleration curves, the integration process tends to have a smoothing effect (note the velocity and displacement curves), resulting in no adverse effects being "felt" by the vehicle body.

Computing Cost

Cost of using the model is directly proportional to the number of time-steps required for the simulation. On the Honeywell G440 the computing time required is approximately two seconds central processor (CP) time per time-step. This means, for simulation of typical cross-country traversal, about 300 to 500 seconds computing time per 100 feet of terrain. A typical charge for CP time for this class of computer is about $.03 per second. Thus cost for simulation of a 1000-foot cross-country would be approximately $90 to $150.

A "number crunching" program such as this would normally be much more economical to run on larger, faster computing systems.

CHAPTER VI: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The results of this study suggest the following conclusions:

A. Vehicle vibrations can be predicted within acceptable error limits using a numerical model (digital simulation) that describes the vehicle as a rigid body connected to rigid axles by springs and viscous dampers.

B. The presence of discontinuities in the representation of riding surface profiles, tire flexure, and suspension springs and dampers does not significantly degrade the performance of the model.

C. The Runge-Kutta-Merson numerical integration method is an effective technique for integrating the differential equations describing the vibratory motions.

D. The numerical integration interval can be dynamically controlled, based on simulation responses and computed integration errors, to provide improved model performance.

1. The period of axle vibration should be considered when establishing a maximum time-step only if axle acceleration itself is of primary importance.
2. Determination of vehicle body accelerations requires that the maximum time-step be limited such that the forward displacement of the vehicle in one time-step does not exceed the tire cross-section height.
3. The minimum time-step is that time-step that corresponds to forward displacement of 0.5 inch.
4. Simulation responses that yield displacements or velocities in excess of physical limits of the system require a time-step reduction.
5. Integration error computed by the Merson formula can be used for the purpose of time-step reduction. An error of 10 in.sec. in vertical velocity of any axle can be used as a maximum allowable integration error.
6. Integration errors computed by the Merson formula can be used for the purpose of increasing the time-step.

If velocity errors for all axles are less than 1 in./sec., a time-step increase is justified.

7. The initial time-step should be the same as the maximum time-step.

Recommendations

It is recommended that

A. The validation of the model be completed by

1. Precise determination of vehicle parameters for a test vehicle.
2. Execution of field tests so that the variables of the tests, i.e. forward velocity, angle of approach, etc., are more precisely controlled, thus eliminating the driver response from the tests.
3. Simulation of the field tests and comparison of the measured and simulated vehicle responses.

B. A study be made to determine the minimum number of tire segments that represents tire flexure to the degree of accuracy necessary to yield acceptable vehicle body accelerations.

C. The model be extended to include simulation of vehicles having other types of suspension systems, e.g., independent suspensions and "walking beam" suspensions.

ABSTRACT

Windell Francis Ingram, Master of Science, 1972

Major: Engineering Mechanics, College of Engineering

Title of Thesis: A Numerical Model of the Ride Dynamics of a Vehicle Using a Segmented Tire Concept

Directed by: Dr. Charles W. Bobbitt

Pages in Thesis: 99 Words in Abstract: 251

ABSTRACT

The purpose of this study was to develop and validate a digital computer simulation of the ride dynamics, including bounce, pitch, and roll, of a general single-hull, solid-axle vehicle with an arbitrary number of axles. The study also included an effort to minimize the execution time and thus the cost of the simulation by providing automatic determination of the optimum numerical integration interval to be used at each point in the simulation.

The vehicle was defined as a rigid body connected to rigid axles by springs and viscous dampers. Tires were described as clusters of radially projecting springs being deflected by a nonyielding riding surface. The suspension springs and dampers were described by force-deflection and force-rate tables, thus allowing nonlinear characteristics to be handled by a piece-wise linear representation with linear interpolation between the points.

The numerical integration method used was the Runge-Kutta-Merson method, which is a fourth-order method. The method requires five

derivative evaluations per integration and provides an estimate of the truncation error that can be used for automatic interval adjustment.

Field tests of a representative vehicle traversing a single obstacle at three different speeds and a cross-country terrain at one speed were simulated. Graphs are presented comparing simulated vehicle responses with field test data provided by the Waterways Experiment Station.

The effects of integration step-size on simulated vehicle responses are illustrated with graphs comparing simulated vehicle responses using several different integration intervals. A discussion of criteria for automatic management of the integration interval is presented.

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APPENDIX

COMPUTER PROGRAM: VEHICLE RIDE DYNAMICS

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$RPC
C             ********************
C
C        GENERALIZED VEHICLE DYNAMICS PROGRAM FOR SINGLE
C        HULL,SOLID AXLE VEHICLES ; USES A SEGMENTED TIRE
C        CONCEPT . THIS PROGRAM IS A MODIFICATION OF A
C        PROGRAM DEVELOPED BY FMC FOR THE WATERWAYS EXPERIMENT
C        STATION.
C
C             ********************
C
C************ DIFINITIONS OF VARIABLES ************
C
C       * NAX       NUMBER OF AXLES
C       * VEL       FORWARD VELOCITY
C       * ACCEL     FORWARD ACCELERATION
C       * BETA      TRAVEL PATH ANGLE
C       * ALPHA     TERRAIN SLOPE ANGLE
C       * DELTIM    INTEGRATION STEP SIZE FOR EQUILIBRIUM COMPUTATION
C       * TSTOP     STOP TIME
C       * HMIN      MINIMUM STEP SIZE FOR INTEGRATION
C       * HSTOP     MAXIMUM TRAVEL DISTANCE FOR VEHICLE
C       * ACGHT     HEIGHT OF AXLE CG AT START OF EQUILIBRIUM COMPUTATIO
C       * BCGHT     HEIGHT OF BODY CG AT START OF EQUILIBRIUM COMPUTATIO
C       * LAX(I)    DISTANCE FROM BODY CG TO AXLE
C       * LSUS(J)   DISTANCE FROM BODY CG TO SUSPENSION
C       * LASUS(J)  DISTANCE FROM AXLE CG TO SUSPENSION
C       * LTIR(J)   DISTANCE FROM AXLE CG TO TIRE
C       * WTB       BODY WEIGHT
C       * WTAX(I)   AXLE WEIGHTS
C       * IBP       BODY PITCH MOMENT OF INERTIA
C       * IBR       BODY ROLL MOMENT OF INERTIA
C       * IAXR      AXLE ROLL MOMENTS OF INERTIA
C       * KSAME(J)  SAMENESS TABLE FOR SUSPENSIONS AND TIRES
C       * KSTN      SUSPENSION-TIRE NUMBER
C       * KSMN      SAMENESS NUMBER
C       * WRAD      WHEEL RADIUS
C       * SREFD(J)  SPRING REFERENCE DISTANCES
C       * TREFD(J)  TIRE REFERENCE DISTANCES
C       * T(K)      TEMPORARY STORAGE FOR CARD DATA
C       * NSEG      NUMBER OF TIRE SEGMENTS  (MAXIMUM 24)
C       * DSEG(I)   SEGMENT CENTERLINE W.R.T. VERTICAL (DEGREES)
C       * SEGK(I)   SEGMENT SPRING CONSTANT
C       * DELTA(K,J),FS(K,J)    PROFILES OF SUSPENSION SPRING FORCE
C       * DELTD(K,J),FD(K,J)    PROFILES OF SUSPENSION DAMPING FORCE
C       * DELD(K,J),FFD(K,J)    PROFILES OF TIRE DAMPING FORCE
C       * WTOT          TOTAL WEIGHT OF TRUCK
C       * FDRIV         HORIZONTAL PROPULSION FORCE
C       * WLAG(I)       LAG DISTANCES OF EACH WHEEL CENTER BEHIND LEAD W
C       * HH            INTEGRATION STEP SIZE
C       * D(1)         XDDB      ACCELERATION OF BODY CG
C       * D(2)         XDB       VELOCITY OF BODY CG
C       * D(3)         THDDB     BODY PITCH ANGLE ACCELERATION
C       * D(4)         THDB      BODY PITCH ANGLE VELOCITY
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C        * D(5)           GDDB      BODY ROLL ANGLE ACCELERATION
C        * D(6)           GDB       BODY ROLL ANGLE VELOCITY
C        * D(7)D(16)      XDDA(I)   ACCELERATION OF AXLE CG
C        * D(17)-D(26)    XDA(I)    VELOCITY OF AXLE CG
C        * D(27)-D(36)    PHDDA(I)  AXLE ROLL ANGLE ACCELERATION
C        * D(37)-D(46)    PHDA(I)   AXLE ROLL ANGLE VELOCITY
C        * D(47)          HDD       HORIZONTAL ACCELERATION OF TRUCK
C        * D(48)          HD        HORIZONTAL VELOCITY OF TRUCK
C        * D(49)          WORKD     TIME DERIVATIVE OF WORK
C        * D(50)          XLDD      COMPUTED HORIZONTAL VELOCITY AT CG
C        * D(51)          TDOT      TIME DERIVATIVE OF TIME
C        *
C        * V(1)           XDB       VELOCITY OF BODY CG
C        * V(2)           XB        POSITION OF BODY CG
C        * V(3)           THDB      BODY PITCH ANGLE VELOCITY
C        * V(4)           THB       BODY PITCH ANGLE
C        * V(5)           GDB       BODY ROLL ANGLE VELOCITY
C        * V(6)           GB        BODY ROLL ANGLE
C        * V(7)-V(16)     XDA(I)    VELOCITY OF AXLE CG
C        * V(17)-V(26)    XA(I)     POSITION OF AXLE CG
C        * V(27)-V(36)    PHDA(I)   AXLE ROLL ANGLE VELOCITY
C        * V(37)-V(46)    PHA(I)    AXLE ROLL ANGLE
C        * V(47)          HD        HORIZONTAL VELOCITY OF TRUCK
C        * V(48)          HP        HORIZONTAL POSITION OF LEAD WHEEL CENTE
C        * V(49)          WORK      WORK (INCH-POUNDS)
C        * V(50)          SB        NOT USED
C        * V(51)          TIME      CURRENT TIME (SECONDS)
C        * NSUS       NUMBER OF SUSPENSIONS
C        * DLS(J)     DEFLECTIONS OF SUSPENSIONS
C        * DLSD(J)    DEFLECTION VELOCITIES OF SUSPENSIONS
C        * DLTD(J)    DEFLECTION VELOCITIES OF TIRES
C        * SPS(J)     SPRING FORCES OF SUSPENSIONS
C        * DMS(J)     DAMPING FORCES OF SUSPENSIONS
C        * DMT(J)     DAMPING FORCES OF TIRES
C        * F(J)       TOTAL SUSPENSION FORCES
C        * FF(J)      TOTAL TIRE FORCES
C        * SUMB       SUM OF FORCES (TEMPORARY)
C        * SUMOM      SUM OF MOMENTS (TEMPORARY)
C        * FHORIZ     SUM OF HORIZONTAL FORCES
C        * HORMOM     SUM OF HORIZONTAL MOMENTS
C        * FFV(I)     TIRE FORCE VECTOR SUM
C        * XWCN       HORIZONTAL POSITION OF WHEEL CENTER
C        * XRET       REAR EXTREME OF TIRE (HORIZONTAL)
C        * XFET       FRONT EXTREME OF TIRE (HORIZONTAL)
C        * CC(I)      POSITION VECTOR OF WHEEL CENTER
C        * TMAG       MAGNITUDE OF RXY VECTORS
C        * AA(I)      TEMPORARY STORAGE
C        * ST         SLOPE OF TERRAIN
C        * SM         SLOPE OF WHEEL SEGMENT
C        * X1,Y1      COORDINATES OF HUB
C        * X2,Y2      COORDINATES OF NEAR TERRAIN STATION
C        * X3,Y3      COORDINATES OF FAR TERRAIN STATION
C        * RXY(I,J)   POINTS ON WHEEL CIRCUMFERENCE
C        * CX(I,J)    COMPONENT OF SEGMENT CENTERLINE
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```
C        * CY(I,J)    COMPONENT OF SEGMENT CENTERLINE
C        * VVV(I)     UNIT VECTOR IN DIRECTION OF TIRE FORCE
C        * VELV(I)    WHEEL CENTER VELOCITY VECTOR
C        * SDLD       TIRE DEFLECTION VELOCITY
C        * TSFOR      TIRE SPRING FORCE
C        * FORVEC(I)  TIRE FORCE VECTOR
C        * FVER(J)    VERTICAL COMPONENT OF TOTAL SUSPENSION FORCE
C        * FLAT(J)    LATERAL COMPONENT OF TOTAL SUSPENSION FORCE
C
C************************************************************
C
      REAL NAX, NPRNT, LSUS, LTIR, LAX, IBP, IBR, IAXR, LASUS
C
C      ..., DIMENSIONS FOR THE FOLLOWING VARIABLES ALLOW.
C      A 10 AXLE VEHICLE . THESE DIMENSIONS MUST NOT BE
C      CHANGED UNLESS THE RELATED EQUIVALENCE STATEMENTS
C      AND THE RELATED SUBSCRIPTING IN THE DERIVITIVE
C      ROUTINE ARE CHANGED ALSO.
C
      COMMON D(51), V(51), VL(51), P1(51), P2(51),
&       P3(51), P4(51), P5(51)
C
C              *******************
C
C      ...DIMENSIONS FOR THE FOLLOWING VARIABLES ALLOW
C      A 3 AXLE VEHICLE. THEY WERE REDUCED TO CONSERVE
C      CORE SPACE. THESE DIMENSIONS MAY BE CHANGED(FOR
C      UP TO 10 AXLES)WITHOUT MAKING ANY OTHER CHANGES.
C
      COMMON LAX(3), LSUS(6), LTIR(6), WTAX(3),
&       IAXR(3), DELTA(25,6), FS(25,6), DELTD(25,6),
&       FD(25,6), DSEG(25,6), SEGK(25,6), DELD(25,6),
&       FFD(25,6), KSAME(6), DLS(6), DLSD(6), DLT(6),
&       DLTD(6), SPS(6), DMS(6), SPT(6), DMT(6),
&       F(6), FF(6), FVER(6), FLAT(6), SREFD(6),
&       TREFD(6), NSEG(6), LASUS(6), WLAG(6), CX(25,6), CY(25,6)
C
C              *******************
C
C      ...DIMENSIONS ON REMAINING VARIABLES ARE NOT
C      A FUNCTION OF THE MAXIMUM NUMBER OF ALLOWABLE
C      AXLES.
C
      COMMON AA(2), CC(2), VELV(2), VVV(2), FORVEC(2),
&       FFV(2), RXY(2,25), THAG(25)
C
      COMMON ICOMNT(24), FHORIZ, XWCN, XRET, XFET,
&       SDLD, TSFOR, TDFOR, SUMFOR, NINVAR, MOTION,
&       NAX, IBR, IBP, HH, ITERR, ITERL, FDRIV,
&       ACCEL, WTOT, WTB, VEL, GG, LUERR, HMIN,
&       HMAX, NNN, JS, ITR, WRAD, IWHL, JSJ, JAJ,
&       INDX, PLTOUT, JAX, NSUS
C
      COMMON STA(50), HGT(50)
```

```
C
      DIMENSION T(8), NMFL(2)
C
      DIMENSION XDDA(10), XDA(10), XA(10), PHDDA(10),
&       PHDA(10), PHA(10)
C
      EQUIVALENCE (XDDB, D(1)), (THDDB, D(3)),
&       (GDDB, D(5)), (XDDA(1), D(7)), (PHDDA(1),
&       D(27)), (HDD, D(47)), (SDDB, D(49)), (TDOT, D(51))
      EQUIVALENCE (XDB, V(1)), (XB, V(2)), (THDB,
&       V(3)), (THB, V(4)), (GDB, V(5)), (GB, V(6)),
&       (XDA(1), V(7)), (XA(1), V(17)), (PHDA(1),
&       V(27)), (PHA(1), V(37)), (HD, V(47)), (HP,
&       V(48)), (SDB, V(49)), (SB, V(50)), (TIME, V(51))
C
      EQUIVALENCE (PLTOUT, NMFL)
C
 100  FORMAT (8E10.0)
 110  FORMAT (16I5)
 120  FORMAT ("0", 4X, "EQUILIBRIUM CONDITIONS" // (5X, 9F12.7))
C
C      ...COMMENTS WILL BE WRITTEN TO THE OUTPUT FILE
C
      PRINT, "COMMENTS"
      READ 130, ICOMNT
 130  FORMAT (24A3)
C
C      ..., INPUT FILE NAME IS PARAMS
C
      CALL OPENF(1, "PARAMS")
C
C      ...ESTABLISH OUTPUT FILE
C
      PRINT, "OUTPUT FILE NAME", * *
      READ 140, PLTOUT
 140  FORMAT (A6)
      WRITE (3) DUMMY
      CALL CLOSEF(3, PLTOUT)
      CALL OPENF(3, PLTOUT)
C
      PRINT, "SPEED OF TRAVERSAL IN IN/SEC", * *
      READ, VEL
      GG = 386.064
      DTR = 3.141592653 / 180.0
C********  READ INPUT DATA  ********
C         ************************
C
      MOTION = 0
C      ..., READ CONSTANTS (DUMMY IS INSERTED FOR VARIABLE
C      FIELDS NO LONGER USED.)
C
      READ (1; 100) DUMMY, ACCEL, BETA, ALPHA
      READ (1; 100) DELTIM
      READ (1; 100) NAX, ACGHT, BCGHT
```

```
      READ (1; 100) WTB, IBP, IBR
      JAX = NAX
      NSUS = 2.0 * NAX
C       ... READ AXLE DISTANCES, SUSPENSION AND TIRE DISTANCES
      READ (1; 100) (LAX(I), I = 1, JAX)
      READ (1; 100) (LSUS(I), I = 1, NSUS)
      READ (1; 100) (LASUS(I), I = 1, NSUS)
      READ (1; 100) (LTIR(I), I = 1, NSUS)
C       ... READ AXLE WEIGHTS AND MOMENTS
      READ (1; 100) (WTAX(I), I = 1, JAX)
      READ (1; 100) (IAXR(I), I = 1, JAX)
C       ... READ REFERENCE DISTANCES
      READ (1; 100) (SREFD(I), I = 1, NSUS)
      READ (1; 100) (TREFD(I), I = 1, NSUS)
      WRAD = TREFD(1)
C       ...READ NUMBER OF SEGMENTS PER TIRE
      READ (1; 110) (NSEG(I), I = 1, NSUS)
C       ... INPUT FORCE PROFILE DATA ...
C
C       ...READ PARAMETER CARD
  150 READ (1; 110) KSTN, KSMN
C       ... TEST FOR FLAG
      IF (KSTN) 330, 330, 160
C       ... SET SAMENESS NUMBER
  160 KSAME(KSTN) = KSMN
C       ... TEST FOR PROFILE DATA
      IF (KSMN) 150, 170, 150
  170 CONTINUE
C       ... READ SUSPENSION SPRING FORCE PROFILE
      IT = 1
  180 READ (1; 100) T
      IF (T(1) + T(3)) 190, 210, 190
  190 DO 200 I = 1, 7, 2
      DELTA(IT,KSTN) = T(I)
      FS(IT,KSTN) = T(I+1)
      IT = IT + 1
  200 CONTINUE
      GO TO 180
C       ... READ SUSPENSION DAMPING FORCE PROFILE
  210 IT = 1
  220 READ (1; 100) T
      IF (T(1) + T(3)) 230, 250, 230
  230 DO 240 I = 1, 7, 2
      DELTD(IT,KSTN) = T(I)
      FD(IT,KSTN) = T(I+1)
      IT = IT + 1
  240 CONTINUE
      GO TO 220
C       ... READ TIRE SEGMENTS AND SPRING CONSTANTS
  250 IT = 1
  260 READ (1; 100) T
      IF (T(1) + T(3)) 270, 290, 270
  270 DO 280 I = 1, 7, 2
      DSEG(IT,KSTN) = T(I)
```

```
      SEGK(IT,KSTN) = T(I+1)
      IT = IT + 1
  280 CONTINUE
      GO TO 260
C       ..., READ TIRE DAMPING FORCE PROFILE
  290 IT = 1
  300 READ (1; 100) T
      IF (T(1) + T(3)) 310, 150, 310
  310 DO 320 I = 1, 7, 2
      DELD(IT,KSTN) = T(I)
      FFD(IT,KSTN) = T(I+1)
      IT = IT + 1
  320 CONTINUE
      GO TO 300
  330 CONTINUE
C       ...,CONVERT FROM DEGREES TO RADIANS
      DO 360 JSJ = 1, NSUS
C       ...,TEST FOR SAMENESS
      IF (KSAME(JSJ)) 360, 340, 360
  340 JS = JSJ
      NNN = NSEG(JSJ)
      DO 350 I = 1, NNN
      DSEG(I,JS) = DSEG(I,JS) * DTR
      CX(I,JS) = SIN(DSEG(I,JS)) * TREFD(JSJ)
      CY(I,JS) =  - COS(DSEG(I,JS)) * TREFD(JSJ)
  350 CONTINUE
  360 CONTINUE
C       ..., READ TERRAIN PROFILE TABLE
      NTERR = 1
      IT = 1
  370 READ (1; 100) T
      IF (T(1) + T(3)) 380, 400, 380
  380 DO 390 I = 1, 7, 2
      STA(IT) = T(I)
      HGT(IT) = T(I+1)
      IT = IT + 1
  390 CONTINUE
      GO TO 370
  400 GO TO (410, 420), NTERR
  410 NTERR = 2
      ITERL = IT - 1
      GO TO 370
  420 ITERR = IT - 1
      ALPHAO = ALPHA
      ALPHAX = ALPHA * DTR
      SINA = SIN(ALPHAX)
      COSA = COS(ALPHAX)
C       ..., ROTATE TERRAIN THROUGH SLOPE ANGLE ALPHA
      DO 440 I = 1, ITERR
      IF (STA(I)) 440, 440, 430
  430 CONTINUE
      XX = STA(I) * COSA - HGT(I) * SINA
      YY = STA(I) * SINA + HGT(I) * COSA
      STA(I) = XX
```

```
      HGT(I) = YY
  440 CONTINUE
      BETAOP = BETA
      BETAX = BETA * DTR
      SINB = SIN(BETAX)
C       ... CONVERT TERRAIN PROFILE TO TRAVEL PATH ANGLE BETA
      DO 450 I = 1, ITERR
      STA(I) = STA(I) / SINB
  450 CONTINUE
C       ... COMPUTE LAG DISTANCES OF WHEEL CENTERS
      TTTD =  - LTIR(1) + LTIR(2)
      IF (COS(BETAX)) 490, 460, 480
  460 DO 470 I = 1, NSUS
  470 WLAG(I) = 0.0
      GO TO 520
  480 WLAG(1) = 0.0
      WLAG(2) = TTTD / (SIN(BETAX) / COS(BETAX))
      GO TO 500
  490 WLAG(2) = 0.0
      WLAG(1) = TTTD / ( - SIN(BETAX) / COS(BETAX))
  500 K = NSUS - 1
      DO 510 I = 3, K, 2
      J = (I + 1) / 2
      WLAG(I) = (LAX(1) - LAX(J)) + WLAG(1)
      WLAG(I+1) = (LAX(1) - LAX(J)) + WLAG(2)
  510 CONTINUE
  520 CONTINUE
C
C          ****************************
C********  EQUILIBRIUM COMPUTATION  ********
C          ****************************
C
C       ... TEST FOR EQUILIBRIUM COMPUTATION
      IF (BCGHT) 550, 530, 550
C       ... READ INITIAL VALUES AT EQUILIBRIUM
  530 READ (1; 100) (V(I), I = 1, NINVAR)
      DO 540 I = 1, NINVAR
      VL(I) = V(I)
      D(I) = 0.
  540 CONTINUE
      GO TO 590
C       ... SET UP INITIAL VALUES FOR EQUILIBRIUM COMPUTATION
  550 DO 560 I = 1, NINVAR
      V(I) = VL(I) = 0.
  560 D(I) = 0.
      D(NINVAR) = 1.0
      XB = BCGHT
      DO 570 I = 1, JAX
  570 XA(I) = ACGHT
C       ANALYSIS
C       ... COMPUTE INITIAL VALUES OF ACCELERATION
      CALL DERIV
C
C       ...THE INITIAL TIME STEP FOR EQUILIBRIUM COMPUTAIONS
```

```
C      IS AN INPUT VALUE, THE MAXIMUM ALLOWED IS
C      8 TIMES THE INITIAL. THE MINIMUM IS 1/8 THE INITIAL.
C
      HH = DELTIM
      HMAX = 8. * HH
      HMIN = .125 * HH
C
C      ... INTEGRATE
  580 CONTINUE
      CALL NTGRAT
C      ... TEST FOR FINISH (EQUILIBRIUM IS ASSUMED TO BE
C      REACHED AFTER SOME ARBIRARY TIME,THE TIME CHOSEN MAY BE
C      CHANGED AS APPROPRIATE FOR THE VEHICLE.
      IF (TIME - 3.) 580, 590, 590
C      ... PRINT EQUILIBRIUM CONDITIONS
  590 CONTINUE
      PRINT 120, (V(I), I = 1, NINVAR)
C      ... TEST FOR CONTINUATION
      PRINT, "STOP TIME,STOP DISTANCE", ' *
      READ, TSTOP, HSTOP
      IF (TSTOP.EQ.0.) CALL EXIT
      MOTION = 1
C        ********************************
C********  DYNAMIC RESPONSE COMPUTATION  ********
C        ********************************
C
C      ... SET UP INITIAL VALUES
      V(NINVAR) = 0.0
      D(NINVAR) = 1.0
      HP = 0.0
      HD = VEL
      VL(47) = HD
C      ... COMPUTE THE INITIAL,MAXIMUM, AND MINIMUM TIME
C      STEPS FOR DYNAMIC RESPONSE AS A FUNCTION OF THE
C      FORWARD VELOCITY OF THE VEHICLE. THE MAXIMUM
C      ALLOWABLE FORWARD DISPLACEMENT PER TIME STEP WAS
C      CHOSEN TO BE 8 INCHES,THE MINIMUM 1/2 INCH.
C
      DO 600 I = 1, 10
      HH = 3. * 2. ** ( - I)
      IF (HH - 8. / VEL) 610, 600, 600
  600 CONTINUE
  610 HMAX = HH
      HMIN = HMAX / 16.
C
      WRITE (3) ICOMNT, VEL, NSEG(1), HMAX, HMIN
      CALL OUTPUT
C
C      ...,NOW READY TO SIMULATE FORWARD MOTION OF
C      THE VEHICLE OVER A TERRAIN OR OBSTACLE COURSE.
C
  620 CALL NTGRAT
      CALL OUTPUT
C      ... TEST FOR FINISH
```

```
      IF (TIME - TSTOP) 630, 640, 640
  630 IF (HP - HSTOP) 620, 640, 640
  640 CONTINUE
      PRINT, "RUN COMPLETED"
      CALL EXIT
      END
C
C             ••••••••••••••••••••   SUBROUTINE NTGRAT
C
C       ...THIS SUBROUTINE USES RUNGE-KUUTA-MERSONS
C       INTEGRATION FORULAE. THE STEP SIZE IS AUTOMATICALLY
C       ADJUSTED BETWEEN THE COMPUTED LIMITS BASED ON THE
C       COMPUTED INTEGRATION ERROR FOR THE VERTICAL
C       MOTION OF THE AXLES. THE STEP SIZE IS ALSO REDUCED
C       IF THE DERIVITIVE ROUTINE RETURNS AN INDICATION
C       THAT DISPLACEMENTS OR VELOCITIES EXCEED THE MAXIMUM
C       VALUES PROVIDED AS VEHICLE PARAMETERS,
C
      SUBROUTINE NTGRAT
  100 LUERR = 1
      HHO3 = HH / 3.
      DO 110 I = 1, NINVAR
      P1(I) = HHO3 • D(I)
  110 V(I) = VL(I) + P1(I)
      CALL DERIV
C       ..., LUERR IS RETURNED AS 2 IF A TABLE LOOK-UP ERROR OCCURS
      GO TO (120, 280), LUERR
  120 CONTINUE
      DO 130 I = 1, NINVAR
      P2(I) = HHO3 • D(I)
  130 V(I) = VL(I) + (P1(I) + P2(I)) • .5
      CALL DERIV
      GO TO (140, 280), LUERR
  140 CONTINUE
      DO 150 I = 1, NINVAR
      P3(I) = HHO3 • D(I)
  150 V(I) = VL(I) + P1(I) • .375 + P3(I) • 1.125
      CALL DERIV
      GO TO (160, 280), LUERR
  160 CONTINUE
      DO 170 I = 1, NINVAR
      P4(I) = HHO3 • D(I)
  170 V(I) = VL(I) + P1(I) • 1.5 - P3(I) • 4.5 + P4(I) • 6.0
      CALL DERIV
      GO TO (180, 280), LUERR
  180 CONTINUE
      DO 190 I = 1, NINVAR
      P5(I) = HHO3 • D(I)
      DU = (P1(I) + P4(I) • 4. + P5(I)) • .5
  190 V(I) = VL(I) + DU
      IDOUBL = 1
      DO 220 I = 1, JAX
      K = I + 6
      ERR = ABS((P1(K) - P3(K) • 4.5 + P4(K) •
```

```
&       4.0 - P5(K) * .5) * .2)
C
C       ...ERROR OF 10 IN/SEC IS THE LIMIT BEYOND WHICH A STEP
C       SIZE REDUCTION OCCURS.
C       ...ERROR OF 1 IN/SEC IS THE LIMIT BELOW WHICH A STEP
C       SIZE INCREASE OCCURS.
C
      IF (ERR - 10.) 200, 200, 280
  200 IF (ERR - 1.) 220, 210, 210
  210 IDOUBL = 0
  220 CONTINUE
      IF (IDOUBL) 250, 250, 230
  230 IF (HH - HMAX) 240, 250, 250
  240 HH = HH * 2.
  250 NXTIME = 5
      CALL DERIV
      GO TO (260, 280), LUERR
  260 CONTINUE
      DO 270 I = 1, NINVAR
      VL(I) = V(I)
  270 CONTINUE
      RETURN
  280 IF (HH - HMIN) 300, 300, 290
  290 HH = HH * .5
      GO TO 100
  300 PRINT, "STEP SIZE LESS THAN MINIMUM,RUN TERMINATED"
      CALL EXIT
      END
C
C               ....................   SUBROUTINE DERIV
C
C       ... THIS SUBROUTINE IS USED BY NTGRAT TO COMPUTE DERIVITIVES
C
      SUBROUTINE DERIV
C
      IF (MOTION) 100, 100, 450
C
C       ... MOTION IS 0 FOR EQUILIBRIUM COMPUTATION , 1 FOR
C       DYNAMIC COMPUTATION.
C
C
C         ..................................................
C........  DERIVATIVE ROUTINE FOR EQUILIBRIUM COMPUTATION  ........
C         ..................................................
C
      IF (MOTION) 100, 100, 450
  100 CONTINUE
C       ... CALCULATE DEFLECTIONS OF TIRES AND SPRINGS
      DO 160 JSJ = 1, NSUS
      JAJ = (JSJ + 1) / 2
C       ... TIRE DEFLECTIONS
      DLT(JSJ) = TREFD(JSJ) - (XA(JAJ) - LTIR(JSJ) * SIN(PHA(JAJ)))
      IF (DLT(JSJ)) 110, 120, 120
  110 DLT(JSJ) = 0.0
```

```
 120 CONTINUE
C       ..., SPRING DEFLECTIONS
      DLS(JSJ) = SREFD(JSJ) - (XB - LAX(JAJ) *
&        SIN(THB) - LSUS(JSJ) * SIN(GB) - XA(JAJ) +
&        LASUS(JSJ) * SIN(PHA(JAJ)))
C       ... CALCULATE DEFLECTION VELOCITIES
      IF (DLT(JSJ)) 130, 130, 140
 130  DLTD(JSJ) = 0.0
      GO TO 150
 140  CONTINUE
      DLTD(JSJ) =  - XDA(JAJ) + LTIR(JSJ) * COS(P
&       HA(JAJ)) * PHDA(JAJ)
 150  CONTINUE
      DLSD(JSJ) =  - XDB + LAX(JAJ) * COS(THB) *
&        THDB + LSUS(JSJ) * COS(GB) * GDB + XDA(JAJ) -
&        LASUS(JSJ) * COS(PHA(JAJ)) * PHDA(JAJ)
 160  CONTINUE
      DO 400 JSJ = 1, NSUS
      JAJ = (JSJ + 1) / 2
C       ...TEST FOR SAMENESS
      IF (KSAME(JSJ)) 170, 180, 170
 170  JS = KSAME(JSJ)
      GO TO 190
 180  JS = JSJ
 190  CONTINUE
      NNN = NSEG(JSJ)
C       ..., LOOK UP SUSPENSION SPRING FORCE
      DO 200 I = 1, 24
      IF (DELTA(I,JS) - DLS(JSJ)) 200, 200, 220
 200  CONTINUE
      PRINT 210, JSJ
 210  FORMAT ( / 5X, "TLU ERROR - SUSPENSION SPRING"I3)
      LUERR = 2
      RETURN
 220  SPS(JSJ) = FS(I-1,JS) + (DLS(JSJ) - DELTA(I-1
&       ,JS)) * (FS(I,JS) - FS(I-1,JS)) / (DELTA(I,JS) -
&        DELTA(I-1,JS))
C       ... LOOK UP SUSPENSION DAMPING FORCE
      DO 230 I = 1, 24
      IF (DELTD(I,JS) - DLSD(JSJ)) 230, 230, 250
 230  CONTINUE
      PRINT 240, JSJ
 240  FORMAT ( / 5X, "TLU ERROR - SUSPENSION DAMP"I3)
      LUERR = 2
      RETURN
 250  DHS(JSJ) = FD(I-1,JS) + (DLSD(JSJ) - DELTD(I-1
&       ,JS)) * (FD(I,JS) - FD(I-1,JS)) / (DELTD(I,JS) -
&        DELTD(I-1,JS))
C       ..., COMPUTE TIRE SPRING FORCE
      SPT(JSJ) = 0.
      IF (DLT(JSJ)) 360, 360, 260
 260  CONTINUE
      CC(1) = TREFD(JSJ)
      CC(2) = XA(JAJ) - LTIR(JSJ) * SIN(PHA(JAJ))
```

```
      X1 = CC(1)
      Y1 = CC(2)
C       ...LOCATE ALL POINTS ALONG TIRE CIRCUMFERENCE
      DO 270 I = 1, NNN
      RXY(1,I) = X1 + CX(I,JS)
      RXY(2,I) = Y1 + CY(I,JS)
  270 CONTINUE
C       ...COMPUTE DEFLECTIONS
      DO 340 I = 1, NNN
      SM = CY(I,JS) / CX(I,JS)
      IF (SM) 290, 280, 290
  280 XX = 1.E35
      GO TO 300
  290 XX = ( - Y1 + SM * X1) / SM
  300 YY = 0.
      X2 = 0.
      X3 = 2. * TREFD(JSJ)
      KKERR = KBTWN(X2,XX,X3)
      GO TO (310, 340), KKERR
  310 KKERT = KBTWN(X1,XX,RXY(1,I))
      GO TO (320, 340), KKERT
  320 KKERU = KBTWN(Y1,YY,RXY(2,I))
      GO TO (330, 340), KKERU
  330 RXY(1,I) = XX
      RXY(2,I) = YY
  340 CONTINUE
      DO 350 I = 1, NNN
      AA(1) = CC(1) - RXY(1,I)
      AA(2) = CC(2) - RXY(2,I)
      TMAG(I) = SQRT(AA(1) * AA(1) + AA(2) * AA(2))
      RXY(2,I) = AA(2) / TMAG(I)
      TMAG(I) = (TREFD(JSJ) - TMAG(I)) * SEGK(I,JS)
      SPT(JSJ) = SPT(JSJ) + TMAG(I) * RXY(2,I)
  350 CONTINUE
  360 CONTINUE
C       ... LOOK UP TIRE DAMPING FORCE
      DO 370 I = 1, 24
      IF (DELD(I,JS) - DLTD(JSJ)) 370, 370, 390
  370 CONTINUE
      PRINT 380, JSJ
  380 FORMAT ( / 5X, "TLU ERROR - TIRE DAMP"I3)
      LUERR = 2
      RETURN
  390 DMT(JSJ) = FFD(I-1,JS) + (DLTD(JSJ) - DELD(I-1
&       ,JS)) * (FFD(I,JS) - FFD(I-1,JS)) / (DELD(I,JS) -
&        DELD(I-1,JS))
C       ... COMPUTE TOTAL FORCES
      F(JSJ) = SPS(JSJ) + DMS(JSJ)
      FF(JSJ) = SPT(JSJ) + DMT(JSJ)
      FLAT(JSJ) = F(JSJ) * SIN(GB)
  400 CONTINUE
C       ... COMPUTE SUM OF SUSPENSION FORCES
      SUMB = 0.0
      DO 410 JSJ = 1, NSUS
```

```
 410 SUMB = SUMB + F(JSJ)
C       ... COMPUTE BODY ACCELERATION AND VELOCITY
      D(1) = (SUMB - WTB) * GG / WTB
      D(2) = V(1)
C       ... COMPUTE SUM OF BODY PITCH MOMENTS
      SUMOM = 0.0
      DO 420 JSJ = 1, NSUS
      JAJ = (JSJ + 1) / 2
 420  SUMOM = SUMOM - LAX(JAJ) * F(JSJ) * COS(THB)
C       ... COMPUTE BODY PITCH ANGULAR ACCELERATION AND VELOCITY
      D(3) = SUMOM / IBP
      D(4) = V(3)
C       ... COMPUTE SUM OF BODY ROLL MOMENTS
      SUMOM = 0.0
      DO 430 JSJ = 1, NSUS
 430  SUMOM = SUMOM - LSUS(JSJ) * COS(GB) * F(JSJ)
C       ... COMPUTE BODY ROLL ANGULAR ACCELERATION AND VELOCITY
      D(5) = SUMOM / IBR
      D(6) = V(5)
C       ... COMPUTE AXLE ACCELERATION AND VELOCITY
      DO 440 JSJ = 1, NSUS, 2
      JAJ = (JSJ + 1) / 2
C       DOH
      D(JAJ+6) = (FF(JSJ) + FF(JSJ+1) - F(JSJ) -
&       F(JSJ+1) - WTAX(JAJ)) * GG / WTAX(JAJ)
      D(JAJ+16) = V(JAJ+6)
C       ... COMPUTE AXLE ROLL ANGLE ACCELERATION AND VELOCITY
      D(JAJ+26) = ( - FF(JSJ) * LTIR(JSJ) * COS(P
&      HA(JAJ)) - FF(JSJ+1) * LTIR(JSJ+1) * COS(PH
&      A(JAJ)) + F(JSJ) * LASUS(JSJ) * COS(PHA(JAJ)) +
&       F(JSJ+1) * LASUS(JSJ+1) * COS(PHA(JAJ))) / IAXR(JAJ)
      D(JAJ+36) = V(JAJ+26)
 440  CONTINUE
      D(47) = 0.0
      D(48) = 0.0
      RETURN
C         ***************************************************************
C********  DERIVATIVE ROUTINE FOR DYNAMIC RESPONSE COMPUTATION  ********
C         ***************************************************************
 450  CONTINUE
      LUERR = 1
      FHORIZ = 0.0
      HORMOM = 0.0
C       ... COMPUTE DEFLECTIONS OF TIRES AND TIRE FORCES
      DO 620 JJJ = 1, NSUS
      JSJ = NSUS - JJJ + 1
C       ...TEST FOR SAMENESS
      IF (KSAME(JSJ)) 460, 470, 460
 460  JS = KSAME(JSJ)
      GO TO 480
 470  JS = JSJ
 480  CONTINUE
C
      JAJ = (JSJ + 1) / 2
```

```
      FFV(1) = 0.0
      FFV(2) = 0.0
      NNN = NSEG(JSJ)
      VVV(1) = 0.
      VVV(2) = 0.
      WRAD = TREFD(JSJ)
C      ... FIND EXTREME REAR OF TIRE AND EXTREME FRONT OF TIRE
      XWCN = HP - WLAG(JSJ)
      XRET = XWCN - WRAD
      XFET = XWCN + WRAD
C      ... LOOK UP TERRAIN COORDINATES
      KLR = JSJ / JAJ
      GO TO (490, 500), KLR
 490  KTR1 = 1
      KTR2 = ITERL
      GO TO 510
 500  KTR1 = ITERL + 1
      KTR2 = ITERR
 510  CONTINUE
      DO 520 ITR = KTR1, KTR2
      IF (XRET - STA(ITR)) 530, 520, 520
 520  CONTINUE
      PRINT, "VEHICLE IS BEYOND LIMITS OF THE"
&      , " TERRAIN,RUN TERMINATED"
      CALL EXIT
 530  CONTINUE
      CALL SEGWHL
      IF (LUERR.EQ.2) RETURN
C      ... COMPUTE WHEEL CENTER VELOCITY VECTOR
      VELV(1) = HD
      VELV(2) = XDA(JAJ) - LTIR(JSJ) * COS(PHA(JAJ)) * PHDA(JAJ)
C      ... COMPUTE TIRE DEFLECTION VELOCITY
      SDLD =  - (VELV(1) * VVV(1) + VELV(2) * VVV(2))
C      ... TEST FOR SAMENESS
      IF (KSAME(JSJ)) 540, 550, 540
 540  JS = KSAME(JSJ)
      GO TO 560
 550  JS = JSJ
 560  CONTINUE
C      ... LOOK UP TIRE DAMPING FORCE
      DO 570 I = 1, 24
      IF (DELD(I,JS) - SDLD) 570, 570, 600
 570  CONTINUE
 580  CONTINUE
      PRINT 590, JSJ, SDLD
      LUERR = 2
      RETURN
 590  FORMAT ( / 5X, "TLU-ERROR - TIRE DAMP"I3, 2X, "SDLD="F10.3)
 600  IF (I - 1) 580, 580, 610
 610  TDFOR = FFD(I-1,JS) + (SDLD - DELD(I-1,JS)) *
&       (FFD(I,JS) - FFD(I-1,JS)) / (DELD(I,JS) - DELD(I-1,JS))
C      ... COMPUTE FORCE VECTOR AND ACCUMULATE
      SUMFOR = TSFOR + TDFOR
      FORVEC(1) = SUMFOR * VVV(1)
```

```
      FORVEC(2) = SUMFOR * VVV(2)
      FFV(1) = FORVEC(1) + FFV(1)
      FFV(2) = FORVEC(2) + FFV(2)
C       ... SET VERTICAL FORCE COMPONENT
      FF(JSJ) = FFV(2)
C       ... ACCUMULATE HORIZONTAL FORCE COMPONENT
      FHORIZ = FHORIZ + FFV(1)
C       ... ACCUMULATE HORIZONTAL MOMENTS
      HORMOM = HORMOM + FFV(1) * (XB - (XA(JAJ) -
     &   SIN(PHA(JAJ)) * LTIR(JSJ)))
  620 CONTINUE
C       ... COMPUTE DEFLECTIONS OF SPRINGS AND SPRING FORCES
      DO 760 JSJ = 1, NSUS
      JAJ = (JSJ + 1) / 2
      IF (KSAME(JSJ)) 630, 640, 630
  630 JS = KSAME(JSJ)
      GO TO 650
  640 JS = JSJ
  650 CONTINUE
C       ... CALCULATE SPRING DEFLECTIONS
      DLS(JSJ) = SREFD(JSJ) - (XB - LAX(JAJ) *
     &   SIN(THB) - LSUS(JSJ) * SIN(GB) - XA(JAJ) +
     &   LASUS(JSJ) * SIN(PHA(JAJ)))
C       ... CALCULATE THE DEFLECTION NELOCITIES
      DLSD(JSJ) =  - XDB + LAX(JAJ) * COS(THB) *
     &   THDB + LSUS(JSJ) * COS(GB) * GDB + XDA(JAJ) -
     &   LASUS(JSJ) * COS(PHA(JAJ)) * PHDA(JAJ)
C       ... LOOK UP SUSPENSION SPRING FORCE
      DO 660 I = 1, 24
      IF (DELTA(I,JS) - DLS(JSJ)) 660, 660, 690
  660 CONTINUE
  670 CONTINUE
      PRINT 680, JSJ
      LUERR = 2
      RETURN
  680 FORMAT ( / 5X, "TLU ERROR - SUSPENSION SPRING"I3)
  690 IF (I - 1) 670, 670, 700
  700 SPS(JSJ) = FS(I-1,JS) + (DLS(JSJ) - DELTA(I-1
     &   ,JS)) * (FS(I,JS) - FS(I-1,JS)) / (DELTA(I,JS) -
     &   DELTA(I-1,JS))
C       ... LOOK UP SUSPENSION DAMPING FORCE
      DO 710 I = 1, 24
      IF (DELTD(I,JS) - DLSD(JSJ)) 710, 710, 740
  710 CONTINUE
  720 CONTINUE
      PRINT 730, JSJ
      LUERR = 2
      RETURN
  730 FORMAT ( / 5X, "TLU ERROR - SUSPENSION DAMP"I3)
  740 IF (I - 1) 720, 720, 750
  750 DMS(JSJ) = FD(I-1,JS) + (DLSD(JSJ) - DELTD(I-1
     &   ,JS)) * (FD(I,JS) - FD(I-1,JS)) / (DELTD(I,JS) -
     &   DELTD(I-1,JS))
C       ... COMPUTE TOTAL FORCES
```

```
      F(JSJ) = SPS(JSJ) + DMS(JSJ)
      FVER(JSJ) = F(JSJ) * COS(GB)
      FLAT(JSJ) = F(JSJ) * SIN(GB)
  760 CONTINUE
C      ... COMPUTE SUM OF SUSPENSION FORCES
      SUMB = 0.0
      DO 770 JSJ = 1, NSUS
  770 SUMB = SUMB + FVER(JSJ)
C      ... COMPUTE BODY ACCELERATION AND VELOCITY
      D(1) = (SUMB - WTB) * GG / WTB
      D(2) = XDB
C      ... COMPUTE SUM OF BODY PITCH MOMENTS
      SUMOM = 0.0
      DO 780 JSJ = 1, NSUS
      JAJ = (JSJ + 1) / 2
  780 SUMOM = SUMOM - LAX(JAJ) * F(JSJ) * COS(THB)
C      PALO
      SUMOM = SUMOM - HORMOM
C      ... COMPUTE BODY PITCH ANGULAR ACCELERATION AND VELOCITY
      D(3) = SUMOM / IBP
      D(4) = THDB
C      ... COMPUTE SUM OF BODY ROLL MOMENTS
      SUMOM = 0.0
      DO 790 JSJ = 1, NSUS
  790 SUMOM = SUMOM - LSUS(JSJ) * F(JSJ) * COS(GB)
C      ... COMPUTE BODY ROLL ANGULAR ACCELERATION AND VELOCITY
      D(5) = SUMOM / IBR
      D(6) = GDB
C      ... COMPUTE AXLE ACCELERATION AND VELOCITY
      DO 800 JSJ = 1, NSUS, 2
      JAJ = (JSJ + 1) / 2
      D(JAJ+6) = (FF(JSJ) + FF(JSJ+1) - F(JSJ) -
     &  F(JSJ+1) - WTAX(JAJ)) * GG / WTAX(JAJ)
      D(JAJ+16) = XDA(JAJ)
C      ... COMPUTE AXLE ROLL ANGLE ACCELERATION AND VELOCITY
      D(JAJ+26) = ( - FF(JSJ) * LTIR(JSJ) * COS(P
     & HA(JAJ)) - FF(JSJ+1) * LTIR(JSJ+1) * COS(PH
     & A(JAJ)) + F(JSJ) * LASUS(JSJ) * COS(PHA(JAJ)) +
     &  F(JSJ+1) * LASUS(JSJ+1) * COS(PHA(JAJ))) / IAXR(JAJ)
      D(JAJ+36) = PHDA(JAJ)
  800 CONTINUE
C      ... COMPUTE HORIZONTAL ACCELERATION OF TRUCK
      D(47) = ACCEL
      D(48) = HD
C      ... EXIT
      RETURN
      END
C
C
C          ********************   SUBROUTINE OUTPUT
C
C
C      ... THIS SUBROUTINE WRITES ACCELERATIONS,VELOCITIES,
C      AND DISPLACEMENTS TO AN OUTPUT FILE AT EACH INTEGRATION
```

```
C     STEP.IT IS CALLED AFTER EACH SUCESSFUL INTEGRATION STEP.
C     IT CAN BE CHANGED TO OUTPUT THESE VARIABLES IN ANY
C     DESIRABLE FORM. ANY OTHER VARIABLES OF INTEREST MAY
C     BE OUTPUT HERE SINCE ALL SIGNIFICANT VARIABLES ARE
C     IN COMMON.
C
      SUBROUTINE OUTPUT
      INDX = INDX + 1
C     ... IS THE FILE FULL
      IF (INDX.LT.126) GO TO 120
C     ...YES,CREATE A NEW FILE
      CALL CLOSEF(3)
      NMFL(2) = NMFL(2) + 1
  100 FORMAT (A6)
      PRINT 110, PLTOUT, V(51)
  110 FORMAT (1X, A6, 1X, "BEGINS AT", 1X, F9.5)
      WRITE (3) DUMMY
      CALL CLOSEF(3, PLTOUT)
      CALL OPENF(3, PLTOUT)
C     ... WRITE THE HEADING INFO AT THE BEGINNING OF EACH FILE,
      WRITE (3) ICOMNT, SPD, NSEG(1), HMAX, HMIN
      INDX = 1
  120 WRITE (3) D, V
      RETURN
      END
C
C
C               ********************   FUNCTION KBTWN
C
C     ... THIS FUNCTION IS USED BY SUBROUTINE SEGWHL TO
C     DETERMINE IF A POINT IS BETWEEN TWO OTHER POINTS.
C
      FUNCTION KBTWN(A, P, B)
      IF (A - P) 110, 120, 100
  100 IF (B - P) 120, 120, 130
C     IS P BETWEEN A AND B
  110 IF (B - P) 130, 120, 120
C     VALID RETURN
  120 KBTWN = 1
      RETURN
C     INVALID RETURN
  130 KBTWN = 2
      RETURN
      END
C
C
C               ********************   SUBROUTINE SEGWHL
C
C     ...THIS SUBROUTINE IS USED BY THE DYNAMIC RESPONSE
C     PORTION OF SUBROUTINE DERIV TO COMPUTE TIRE DEFLECTION
C     FORCE VECTORS USING THE SEGMENTED TIRE CONCEPT.
C
      SUBROUTINE SEGWHL
      CC(1) = XWCN - STA(ITR-1)
```

```
      CC(2) = XA(JAJ) - LTIR(JSJ) * SIN(PHA(JAJ))
      X1 = CC(1)
      Y1 = CC(2)
C      ...LOCATE ALL POINTS ALONG TIRE CIRCUMFERENCE
      DO 100 I = 1, NNN
      RXY(1,I) = X1 + CX(I,JS)
      RXY(2,I) = Y1 + CY(I,JS)
  100 CONTINUE
C      ...INTERSECT SEGMENT CENTERS WITH TERRAIN
      MOLE = 1
      DO 220 I = 1, NNN
      DMIN = 1.E10
      SM = CY(I,JS) / CX(I,JS)
      KTR = ITR
      X2 = 0.
      Y2 = HGT(KTR-1)
  110 CONTINUE
      X3 = STA(KTR) - STA(ITR-1)
      Y3 = HGT(KTR)
      ST = (Y2 - Y3) / (X2 - X3)
      IF (SM - ST) 130, 120, 130
  120 XX = 1.E35
      GO TO 140
  130 XX = (Y2 - Y1 + SM * X1 - ST * X2) / (SM - ST)
  140 YY = ST * (XX - X2) + Y2
      KERR = KBTWN(X2,XX,X3)
      GO TO (150, 180), KERR
  150 KERS = KBTWN(Y2,YY,Y3)
      GO TO (160, 180), KERS
  160 KERT = KBTWN(X1,XX,RXY(1,I))
      GO TO (170, 180), KERT
  170 KERU = KBTWN(Y1,YY,RXY(2,I))
      GO TO (200, 180), KERU
  180 IF (XFET - STA(KTR)) 220, 220, 190
  190 KTR = KTR + 1
      X2 = X3
      Y2 = Y3
      GO TO 110
  200 DMIN1 = SQRT((XX-X1) * (XX - X1) + (YY - Y1) * (YY - Y1))
      IF (DMIN1 - DMIN) 210, 210, 180
  210 DMIN = DMIN1
      RXY(1,I) = XX
      RXY(2,I) = YY
      MOLE = 0
      GO TO 180
  220 CONTINUE
      IF (MOLE) 290, 290, 230
  230 CONTINUE
      KTR = ITR
  240 IF (STA(KTR) - XWCN) 250, 260, 260
  250 KTR = KTR + 1
      GO TO 240
  260 TEMP1 = HGT(KTR-1)
      TEMP2 = STA(KTR-1)
```

```
      PBHUB = TEMP1 + ((HGT(KTR) - TEMP1) / (STA(
&      KTR) - TEMP2)) * (XWCN - TEMP2)
      IF (Y1 - PBHUB) 270, 270, 290
 270  PRINT 280, JSJ, XWCN
 280  FORMAT ( // 1X, "CENTER OF WHEEL NUMBER",
&       I3, 1X, "IS BELOW THE TERRAIN SURFACE AT XWCN =", F12.2)
      LUERR = 2
      RETURN
 290  CONTINUE
C
      DO 300 I = 1, NNN
      RXY(1,I) = CC(1) - RXY(1,I)
      RXY(2,I) = CC(2) - RXY(2,I)
 300  CONTINUE
C       ...NORMALIZE RXY AND LET TMAG BE MAGNITUDE
      DO 310 I = 1, NNN
      AA(1) = RXY(1,I)
      AA(2) = RXY(2,I)
      TMAG(I) = SQRT(AA(1) * AA(1) + AA(2) * AA(2))
      RXY(1,I) = AA(1) / TMAG(I)
      RXY(2,I) = AA(2) / TMAG(I)
 310  CONTINUE
C       ...COMPUTE FORCE FOR EACH SEGMENT
      DO 320 I = 1, NNN
      TMAG(I) = (WRAD - TMAG(I)) * SEGK(I,JS)
 320  CONTINUE
C       ...COMPUTE COMPOSITE FORCE VECTOR
      DO 330 I = 1, NNN
      VVV(1) = VVV(1) + TMAG(I) * RXY(1,I)
      VVV(2) = VVV(2) + TMAG(I) * RXY(2,I)
 330  CONTINUE
C       ...COMPUTE FORCE MAGNITUDE, TSFOR
      TSFOR = SQRT(VVV(1) * VVV(1) + VVV(2) * VVV(2))
      IF (TSFOR) 350, 350, 340
 340  CONTINUE
      VVV(1) = VVV(1) / TSFOR
      VVV(2) = VVV(2) / TSFOR
      GO TO 360
 350  VVV(1) = 0.
      VVV(2) = 1.
 360  CONTINUE
      RETURN
      END
```

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13. ABSTRACT
The purpose of this study was to develop and validate a digital computer simulation of the ride dynamics, including bounce, pitch, and roll, of a general single-hull, solid-axle vehicle with an arbitrary number of axles. The study also included an effort to minimize the execution time and thus the cost of the simulation by providing automatic determination of the optimum numerical integration interval to be used at each point in the simulation. The vehicle was defined as a rigid body connected to rigid axles by springs and viscous dampers. Tires were described as clusters of radially projecting springs being deflected by a nonyielding riding surface. The suspension springs and dampers were described by force-deflection and force-rate tables, thus allowing non-linear characteristics to be handled by a piece-wise linear representation with linear interpolation between the points. The numerical integration method used was the Runge-Kutta-Merson method, which is a fourth-order method. The method requires five derivative evaluations per integration and provides an estimate of the truncation error that can be used for automatic interval adjustment. Field tests of a representative vehicle traversing a single obstacle at three different speeds and a cross-country terrain at one speed were simulated. Graphs are presented comparing simulated vehicle responses with field test data provided by the Waterways Experiment Station. The effects of integration step-size on simulated vehicle responses are illustrated with graphs comparing simulated vehicle responses using several different integration intervals. A discussion of criteria for automatic management of the integration interval is presented.

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